# CHARACTERISTICS
# OF
# HEAVY METAL CHORD STRUCTURES

## Their Acoustic and Modal Construction, and Relation to Modal and Tonal Context

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## Contents

# 1. Introduction

Heavy metal music has been under little academic scrutiny, although some research has been done from cultural or sociological standpoints (e.g. Walser 1993, Berger 1999). Unlike its precedents, the aim of this study is to provide a music analytical perspective to the musical genre, or genres, known as heavy metal. The main object is to illuminate the melodic and harmonic features that are the most significant to this music genre. Furthermore, new tools for music analysis are presented throughout the course of the study. These analytical tools are based on music theory and acoustics in such a way that the starting point has been the music as it sounds. That is to say, the analyses presented here are built to reflect the characteristics of the music. This definitively contrasts with approaches which take a given theory or an analytic model as a staring point and try to fit the music accordingly – this can easily happen if we look at heavy metal music through the theories of major/minor tonal music. By using musical features as a starting point, this study aims particularly to make justice to the music itself.

Rock music has mainly been studied from perspectives other than that of the music – rock music has generally been studied from historical (e.g. Stuessy 1994) or sociological standpoint (e.g. Frith 1983), but rarely from a purely musicological standpoint. It is not until quite recent years that the musicological approach has been extensively applied to rock music by scholars like Walter Everett (2000a, 2001), Allan Moore (1992, 1995, 2001, 2003), and John Covach (1997, and Covach & Boone 1997). Although, rock music is beginning to gain wider acceptance in the field of music theory and analysis, there still are very few such studies on heavy metal. Harris M. Berger (1999) has studied musical experience of death metal musicians by ethnographic case studies, and tried to find relationships between musical activity and large-scale political and economical conditions, and "to provide new insights into the interpretation of musical experience and the role of expressive culture in society" (Berger 1999: 2-3). Robert Walser (1993: xiv) draws "attention to the *music* of heavy metal, in the ways that are both textually specific and culturally grounded." Besides the historical and cultural aspects of heavy metal, he offers many important insights into the music – especially on heavy metal's appropriation of classical music (Walser 1993: 57-107;

published also in Walser 1992). While both Berger and Walser incorporate music analysis in their work, their main concerns are somewhat socio-cultural; musical analysis itself is, although important, of secondary concern.

While I am not aiming to undermine the importance of these (or any) kinds of approaches, it seems to me that much basic work with musical structures has to be done. This is to know first how the music works before one can thoroughly explore, say, "musical experience," "musical meanings," or other fields involving music. In Allan Moore's (2001: 6) words: "Although the sounds of rock cannot, ultimately, be divorced from their [social and cultural] setting, they must be loosely separated in the interim, if the listening act is to receive adequate attention in any discussion of the cultural practices of rock." The study in hand is explicitly concerned with music analysis, and through music analysis paving the way for the forthcoming studies that may have wider perspectives.

In heavy metal music, there are many features in musical structures similar to the so-called "common practise" tonal music.[1] However, one can find many musical characteristics that derive from other sources as well. Considering the melodic and harmonic aspects specific to heavy metal, there are at least four overlapping fields that have to be covered. These fields include 1) modality, 2) voice leading, 3) chord structures in relation to tonal and modal context, and 4) functional tonal hierarchies. Because the issues presented here result in lengthy discussions due to their novelty, the functional tonal hierarchies in heavy metal have been ruled out of this study. However, they will be covered in my future studies. The main concerns here are with the acoustic and modal characteristics of heavy metal chords. Chapter 2 presents the acoustic features that interlock with the practises of chord constructing. Furthermore, some extra attention is paid to a specific chord that is definitive to the genre – namely, the "power chord" that in its simplest form consists of an interval of a perfect fifth or a fourth. This relates closely to the issues presented in Chapter 3, where the position of the chord root in heavy metal and the problems with its definitions are discussed. Chapter 4 covers the most important traditions that have formed the basic scalar elements of heavy metal – the blues-derived pentatonicism, modal scales, and the common-practise tonal music.

[1] Here, "common practise tonal music" is used to refer to the classical-romantic period in the Western art music.

The appearances of these traditions in heavy metal are discussed with some notions to the voice leading practises. Chapter 5 draws the previous chapters together in suggesting a categorizing system for heavy metal chords according to their relations to modal and tonal contexts.

The analyses made here have a strong emphasis on the vertical appearances of heavy metal chords. By no means is this meant to undermine the horizontal aspects of heavy metal harmony. On the contrary, the analyses made here look forward to creating a basis for the more horizontally oriented analyses. After all, a thorough investigation of the construction of individual chord structures is a necessary first step for more elaborate studies.

The musical examples presented in this work start from the ancestors of heavy metal in the late 1960's and go through the 1970's to the mid 1980's – now considered as the golden era of heavy metal (Walser 2004a). Furthermore, there are some more contemporary examples, too (e.g. the analysis of Mercyful Fate's "Banshee" in Chapter 5). As the forefathers of heavy metal bands and artists like Jimi Hendrix, Cream and The Who are taken into an account – i.e. those, who had a profound influence on the formation and the development of the genre in the following decades. As a musical genre, heavy metal was defined in the early seventies by such bands as Led Zeppelin, Deep Purple, and Black Sabbath. From then on, this study is mainly concerned with bands ranging from the so-called "new wave of British heavy metal" of the mid and late seventies (bands like Judas Priest, Iron Maiden, and Motörhead) to the mid eighties with bands like Rainbow, Dio, and Ozzy Osbourne. Although the bands in question here are mainly of British origin, some mention is also made to the ones from other parts of Europe (Accept from Germany, Mercyful Fate from Denmark), the United States of America (Metallica), and Australia (AC/DC). The heavy emphasis on the British is grounded on the unquestionably overriding effect that British metal has had on the formation of the genre at least until the mid eighties – this has been stated in previous studies (see, e.g. Moore 2004; Walser 2004a; 1993: 9-12). In spite of the necessity of dealing with a certain limited time span, the study in hand is not a historical, but rather a theoretical and analytical one. The musical examples are meant to illuminate the theoretical and analytical points made throughout the work.

## 1.1. Heavy Metal as a Musical Genre – Development and Boundaries

According to Robert Walser (1993: 7), "the term 'heavy metal' has been applied to popular music since the late 1960's." Usually scholars refer to Steppenwolf's "Born to be wild" (1967) as the first rock song to popularize the term[2] (Moore 2004; Walser 1993:8). Hard rock bands of the late 1960's – for example, The Who, Cream, The Yardbirds and The Jeff Beck Group from Britain, and Jimi Hendrix from America – are nowadays widely recognized as having had major influence on the formation of heavy metal over the following decades (Walser 2004a; 1993: 9; Obrecht 1984: 8; "Heavy Metal" 2004). Those bands "developed a more distorted guitar sound and heavier drums and bass that led to separation of heavy metal from other blues-based rock" (Walser 2004a). Jimi Hendrix is sometimes credited for releasing "the first real heavy metal hit" "Purple Haze" in 1967 (Walser 1993: 9), and Led Zeppelin is more than often acknowledged as the "the first true metal band" or as the founders of heavy metal.[3] Their album *Led Zeppelin II* (1970) codified the new genre, together with Black Sabbath's *Paranoid* (1970) and Deep Purple's *Deep Purple in Rock* (1970) (Walser 2004a; 1993: 10). During the 1970's, several followers of these three bands (many of them from America) modified heavy metal into more accessible form ("Heavy Metal" 2004). In general, this meant simpler and more pre-arranged structures in contrast to early heavy metal's often complex and improvised forms – this can be clarified by comparing, for instance, the music of Deep Purple or Black Sabbath to that of Kiss or AC/DC. Still, at the time there were significant contributions made to the popularity of the genre: "performers such as AC/DC, Judas Priest, Kiss and Alice Cooper toured incessantly with elaborate stage shows, building a fan base for an internationally-successful style" (Walser 2004a). However, "popularity waned at the end of the decade, but the early 1980s brought the 'new wave' of British heavy metal to revive the genre" (ibid.). The new wave of British heavy metal (sometimes abbreviated as NWOBHM[4]) included bands like Judas Priest, Iron Maiden and Motörhead ("Heavy Metal" 2004). Robert Walser (2004) sums up the next decades in the following:

[2] The song lyrics contain a line "heavy metal thunder".
[3] See, e.g. "Heavy Metal" 2004, or "Led Zeppelin" on Russ 2004.
[4] See, e.g. Russ 2004: "What does NWOBHM stand for?" on the FAQ section of *The BNR Metal Pages*, http://www.bnrmetal.com/pages/faq.htm (accessed May 7, 2004).

> The 1980s brought on the one hand a wave of gender-bending, spectacular 'glam' metal from bands such as Poison and Mötley Crüe, and, on the other hand, the widespread adaptation of chord progressions and virtuosic practices from 18th-century European models, especially Bach and Vivaldi, by influential guitarists such as Van Halen, Randy Rhoads [from Ozzy Osbourne] and Yngwie Malmsteen. Heavy metal was the most popular genre of rock music worldwide during this decade, even as harder underground styles developed in opposition to the pop-oriented metal of groups such as Bon Jovi. Metallica was the most influential band in trash metal. In the 1990s, Metallica, Van Halen, Ozzy Osbourne and other veteran performers continued their success, but the term heavy metal was less often used to distinguish them from the rock mainstream. New groups such as Soundgarden, Korn and Rob Zombie continued the heavy metal tradition in some ways, but were not particularly concerned with claiming the genre label, which had lost much of its prestige.

Still unlike Walser's notion above, the nineties saw the rise of various extreme heavy metal subgenres within which the bands usually claimed not only to be metal, but even "heavier" metal than anyone before. The subgenres that are usually called "black" or "death metal" include such bands as Bathory, Mayhem, Emperor, and Dimmu Borgir. Black and death metal bands derived their music mainly from trash metal (many times adding synthesizers or other keyboards to their ensembles), but owe as much to Black Sabbath.[5]

Furthermore, in the past few years many heavy metal bands that were popular in the eighties have seen a revival in their success and have possibly re-launched the "classic" form of heavy metal to a new rise. This can be seen in the increase in record sales and substantial touring often conducted on the massive scale of the 1980's by bands like Iron Maiden, Ozzy Osbourne, Deep Purple, and Black Sabbath. One reason for this might be that heavy metal fans of the eighties, now in their thirties and forties, have both enough money and enough nostalgia to support their favourite bands.

*On the Boundaries of the Genre*

According to Harris M. Berger (1999: 312) heavy metal is "a group of musical genres [-- that --] often employs distorted guitar timbres, individual displays of virtuosity, and complex song forms." However, like most musical genres, the boundaries of heavy

[5] See, e.g. "Death Metal/Black Metal" on *AMG All Music Guide*, http://www.allmusic.com/cg/amg.dll?p=amg&uid=UIDSUB04040516130224195 1&sql=C384 (accessed May 16, 2004).

metal as a category are quite vague. In his in-depth study on heavy metal, Robert Walser (1993:4) presents "heavy metal" as "a term that is constantly debated and contested, primarily among fans but also in dialogue with musicians, commercial marketing strategists, and outside critics and censors." It really seems that what is heavy metal for some is not heavy metal for others. This has a lot to do at least with the age and musical preferences of an individual. For instance, most people of my generation (born around the early 1970's) would consider Deep Purple as a heavy metal band, but in case of Led Zeppelin opinions would vary much more. Representatives of a younger generation of metal fans (followers of bands like Emperor or Dimmu Borgir) would not probably call either of these "heavy metal." They tend to dismiss Deep Purple or Led Zeppelin (as the recent comments of my undergraduate students have show) as "some old rock acts." Regarding the effect personal listening habits or musical style preferences have on categorising, Walser (1993:6) quotes Iron Maiden's lead singer Bruce Dickinson from an interview for *Musician* magazine[6]: "I wouldn't call UFO a heavy metal band, but if you happen to be a fan of Human League, they probably are. Moreover, if you're a fan of Motörhead, UFO aren't heavy metal. [-- heavy metal is] a category." The categorisation is not axiomatic even for artists themselves – bands that are generally considered heavy metal, may sometimes renounce belonging to that category while others claim to be – and take pride being – the definitive members of the genre. For instance, Led Zeppelin and AC/DC have rejected the term whereas, for example, Judas Priest treasure it. (Walser 1993: 6.)

In answer to the question whether particular bands are, or are not, heavy metal, Brian Russ (2004), a metal fan and the webmaster of a huge heavy metal Internet encyclopaedia *BNR Metal Pages*, states the following:

> This is commonly asked about bands such as Korn, Nirvana, Papa Roach, and others, that may not fit the classic metal mold. This is a source of endless debate -- while there are some bands that everyone can agree are metal, there are others that are less clear. Is nu-metal metal? What about glam/hair bands -- are they metal? It's not an easy question to answer. I have tried to include bands that, if not obviously metal, have sufficient metallic qualities or are of interest to metal fans.

[6] *Musician*, September 1984, p. 53 (Walser 1993: 6).

Another subject of endless dispute among fans and scholars is the question as to whether a particular band is "heavy metal" or "hard rock." Moore (2004) states that hard rock is "an imprecise term, partly co-extensive with heavy metal." In fact, in many cases (including this study) those two terms are treated as synonyms. A perfect example on this is presented in the title of a pedagogic article by Dave Whitehill (1989): "A Heavy Metal Primer – The Fundamentals of Hard Rock Technique."

Heavy metal is often seen as being derived from hard rock as its subgenre (Walser 2004a), but as in almost all genre-labelling we are usually forced to refer to elements outside the music while attempting to establish a difference between heavy metal and hard rock. For example, Moore (2004) basically relies on the lyrics: "The subject matter of the [hard rock] songs emphasizes a misogynistic, macho sexuality and an unfocussed but often environmentally aware liberal politics. Hard rock, however, avoids heavy metal's leaning towards madness, violence and the occult." However true this may be, the distinction is made by other than musical terms. Actually, it is quite possible that the distinction cannot be made by musical terms alone. Elsewhere Moore (2001:147-151) has pursued to find some musically significant distinguishing features, asking the profound question "to what extent do the subtle differences between different brands of metal (about which fanatics make great play) equate to differences of musical style [--]?" (ibid: 151). In spite of several important findings, some conclusions seem simplistic and contradictory – perhaps because Moore emphasizes the role of heavy metal as a subgenre of hard rock. For example, Deep Purple is said to be hard rock and not heavy metal, although there is no obvious reason for this. Apparently, this labelling is done on the grounds that hard rock tends to favour an organ instead of a second guitar (ibid: 149). However, the underlying reason as to why the favouring of an organ over a second guitar should be the musical definition for hard rock, appears to be that in Deep Purple there is an organ and not a second guitar (ibid.). So it seems that Moore has first decided that Deep Purple is hard rock and the musical "factor[s] that serve to identify them" (ibid: 148) are then used only to support this pre-determined view. In fact, it is hard to find an unambiguous distinction between hard rock and heavy metal, and many bands (such as Deep Purple) are frequently put under both labels.

Nevertheless, this study follows Moore's suggestion (ibid.) to treat "hard rock" and "heavy metal" as points of a style continuum. However, this is done without trying to point out some significant differentiation. In fact, those terms are very much in parallel: if someone called something hard rock and something heavy metal, the next writer would probably do the exact opposite. Thus, all the music studied here is in some way put under the label "heavy metal." As stated above, the objective of this study is not to define exactly which bands are, or are not, heavy metal, but rather to explore some common melodic-harmonic features of the music that at some point of time has been called heavy metal (together with its main influences and numerous subgenres). Only by doing this we might, some day, be able to tell, what are the basic *musical* features distinguishing heavy metal from other styles. Still, the question remains as to whether this kind of strict categorisation is ever possible or relevant at all. As stated before, the usage of the term (and the terms) seems to be quite a personal matter varying from one individual to another. Here, because of this, the term is understood and used in a very broad sense. Thus, in the text, musical examples from commonly accepted heavy metal canon of the 1970's and 1980's coincide with the pre-heavy music of The Who, Jimi Hendrix and Cream of the late 1960's.

## 1.2. A Notion on Transcriptions and Reductive Notation

Since the commercially available notations and the guitar tablatures available on the Internet usually have a considerable amount of errors all of the transcriptions here are mine. Because the analyst is forced to make choices on which aspects to include in the transcriptions, they are inevitably subjective representatives of the music (see, e.g. Martin 1996: 5-6). Furthermore, some musical features are impossible to describe with the Western notation system. The insufficiencies of the notation system are more visible when there are features in the music that are alien to Western art music (e.g. blue notes, cf. Titon 1977:154-157). The music is inevitably filtered through the conventions of Western art music and the transcriptions may not carry all the significant features of the music (see, e.g. Seeger 1977: 170). However, Charles Seeger's (1977: 168-171) striving towards an objective description of the music also veers towards results of similar kind – in the end, even the highly objective descriptions (e.g. electronic) will take on fundamentally subjective qualities when they are put

through an individual's (subjective and culturally grounded) interpretation. This problem of subjectivity is not enough to demolish altogether the value of musical transcription in the analysis of musical structures. It is necessary to use language common enough to both musicians and musicologists if we are to mediate any musical features between them. However, due to the necessary selectiveness of the transcription process and the insufficiencies of our notation system, aural analysis of the music should always be combined with analysis on paper.

Furthermore, whenever possible I have used analytic reductions. The most famous scholar using reduction notation was without question Heinrich Schenker (1868-1935) (see, e.g. Schenker 1979). Nevertheless, the reductions applied to the music here are *not* Schenkerian. Here, the reductions are much more used in the spirit of, for example, Leonard B. Meyer (1956: 54, 98) – to reduce a possibly wide musical score to a more simplified form. Meyer's use of reductions is not to promote a musical style as is often done thanks, not so much to Schenker himself, but to his disciples and followers (see, e.g. Covach 1997: 123-124). The reason for using reductions here is simply a matter of convenience – with them I have pointed out certain factors in the music that I think are the most relevant for understanding the musical characteristics under discussion, and have left out the ones that are not so crucial. Of course, this could easily be disputed and regarded as subjectivity, thus I invite the enthusiastic reader to listen to the music while reading this study and to draw conclusions and reductions of their own.

# 2. The Acoustics of the Loud and Distorted Chords

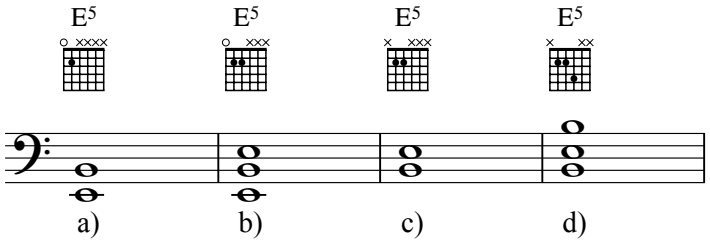
This chapter deals with some typical chord structures used in heavy metal and their acoustic qualities. Some special attention is given to the so-called "power chord."

The power chord is an especially normative musical feature for heavy metal. According to Walser (1993: 2) "the power chord is used by all of the bands that are ever called heavy metal and, until heavy metal's enormous influence on other musical genres in the late 1980s, by comparatively few musicians outside the genre." Pete Townshend of The Who is often credited with being the first one to use power chords extensively for musical purposes (e.g. Walser 1993: 77; 2004b; Murray 2003; McEvoy 2000), although The Kinks had released the first hit song built around power chords in 1964, "You really got me" (Walser 1993: 9).[7] However, the origins or the inventors of the *term* are not known. In all probability, music journalists are responsible for the term's wide acceptance nowadays – by the 1980's at the very latest it began to appear widely in guitarists' magazines in parallel with heavy metal's rise to the popular mainstream.[8]

In this study the term "power chord" refers to a chord that consists only of an interval of the fifth or the fourth with possible octave doublings – as it is generally used by heavy metal musicians (see, e.g. Berger 1999: 313; Whitehill 1989: 122; Marshall 1997: 124, 127). In chord symbols, power chords are indicated with a figure "5" (referring to the interval of the fifth) attached to the letter indicating the chord root – for example, $E^5$ consists of only E and B. Power chords have no thirds. In example 2.1, there is an $E^5$ power chord in four typical positions for guitar – a, b and c being the most common. The most basic form consists of a perfect fifth, or that with the root doubled in octave (examples 2.1a and 2.1b). Example 2.1c presents the inversion of the first one – for instance, this form is used in the famous riff in Deep Purple's "Smoke on the water" (1972). The last one (example 2.1d) is the same as the previous with its lowest note doubled in octave – this one is frequently used in more contemporary

[7] Dave Davies of The Kinks emphasizes the The Kinks' influence on The Who: "Ray [Davies, the songwriter of The Kinks] has influenced a lot of Townshend's compositions" (Forte 1977: 53).
[8] See, e.g. *Guitar Player*, *Guitar World* and *Guitar for the Practicing Musician* of the 1980's.

metal, e.g. Mercyful Fate's "Banshee" (1998), but can be found as early as in Jimi Hendrix's "The wind cries Mary" (1983 [1967]).



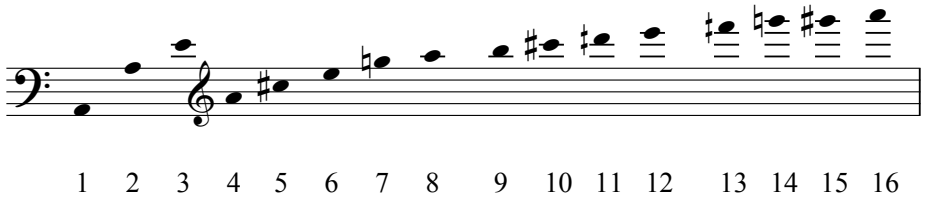

Example 2.1. $E^5$ power chord in some typical guitar positions.

Although the power chord is apparently made only of a single interval, its sound is remarkably more complex. Acoustically this is due to the heavy distortion and loud volumes that emphasize harmonics and generate resultant tones (see, e.g. Walser 1993: 43). "An effect of both distortion and volume, resultant tones are created by the acoustic combination of two notes" (ibid.). However, before entering further discussion on acoustics, a notion on pitch nomenclature should be made here. As it has been done by Lloyd & Rastall (2004), *italic* letters are used to denote specific pitches in various octaves, and non-italic capital letters denote general pitch classes. In differentiating the various octaves, the Helmholtz system is applied throughout this study: the octaves above the middle C are designated as *c'* (middle C) – *c''* – *c'''* – etc., and the octaves below are designated as *c* (one below middle C) – *C* – *C'* – *C''* – etc. (ibid.).

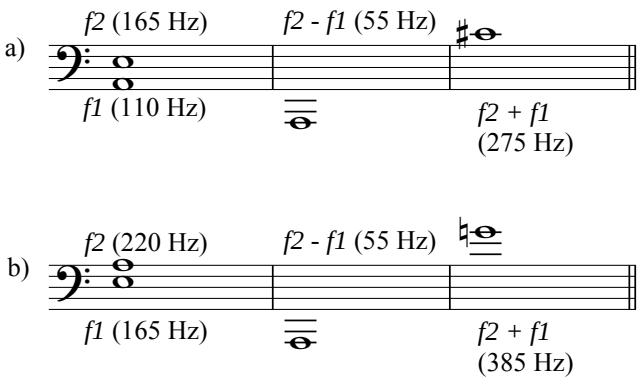
Regarding the acoustics of the power chord (and other chord structures), there are two main phenomena to be covered: 1) the harmonic series, and 2) the combination tones (the difference tones and the summation tones). Every note played on the guitar (or produced by any other instrument) consists not only of the fundamental tone, but of a number of other tones, too. Every note consists of harmonic partials, or components, in frequencies that are the multiples of the fundamental tone (i.e. the first harmonic). "The note *A* sung by a bass, for example, will contain components with frequencies 110, 220, 330, 440, 550, 660 Hz etc." (Lindley et al. 2003). This concept is known as the harmonic series. Usually, the harmonics gradually weaken in ascending order – the higher numbered a harmonic the weaker the sound (Pierce 1999b: 277). However, the behaviour of the harmonics is quite difficult to predict, especially with distorted sound. For example, there is reason to believe that the human ear can compensate the low-

level sounds (Mathews 1999: 9). Furthermore, it has been shown that different instruments emphasize different harmonics and other features, such as the sound pressure's effect on the sound spectra (see, e.g. Pierce 1999a: 65-66; Rossing 1990: 190, 230-231, 270). Furthermore, different types of distortion and amplifiers give rise to different harmonics. For example, power amplifiers usually emphasize only odd-numbered harmonics while pre-amplifiers treat all harmonics as somewhat equal (see, e.g. Lassfolk 1996). However, as the lowest harmonics are closest to the fundamental frequencies it can be postulated that they form the most audible tones and thus audible harmonies. Although the examples below concentrate on just a few of the first harmonics, they give enough evidence on the complexity of heavy metal guitar sound.

For simplicity, all of the following examples in this chapter are given in proportion to A and its higher and lower octaves. For example, striking the lowest A string on the guitar (110 Hz) will result the harmonics shown in example 2.2 – this example will be significant to the following discussion since the harmonics are given extra strength by distortion. In theory, the number of harmonic partials is infinite, yet the series here is presented only with the first sixteen harmonics.




Example 2.2. The first 16 partials of the harmonic series of *A* (110 Hz) (an approximate, equal-tempered notation). The frequency of a harmonic is 110 Hz multiplied by the harmonic's number.

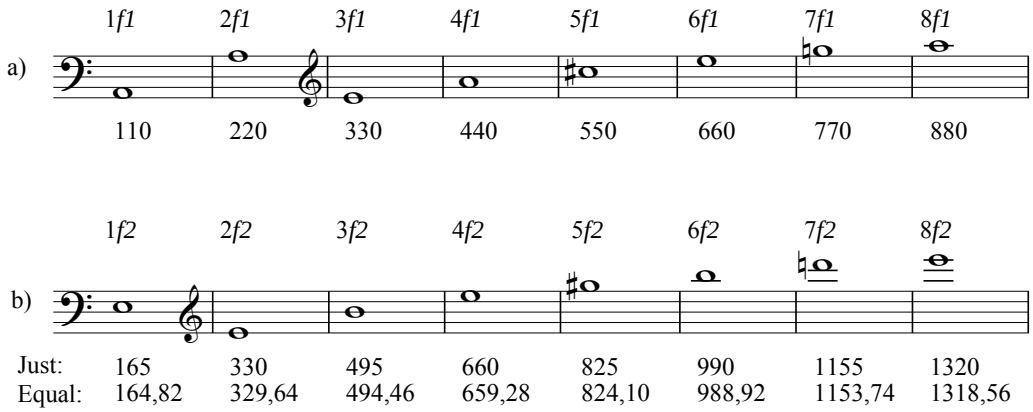



Example 2.3. The simplest difference and summation tones of the fifth *A*-*e* (a) and the fourth *e*-*a* (b).

A combination tone is "a sound that may be heard when two loud musical tones are sounded together but is not present when either of the tones is sounded separately" (Greated 2003b). The discussion on combination tones is easy to start with single frequencies that have no partials sounding with them. Theoretically, if we have frequencies $f_1$ and $f_2$, the simplest resultant combination tones are $f_2 - f_1$ (the difference tone) and $f_1 + f_2$ (the summation tone) (see, e.g. Rossing 1990: 151, 153, Benade 1976: 256-257). For instance, as shown in example 2.3a, the sounding frequencies 110 Hz ($f_1$) and 165 Hz ($f_2$) (the justly intoned[9] equivalents of *A* and *e* in our notational system) result in the combination tones of 55 Hz ($f_2 - f_1$) and 275 Hz ($f_1 + f_2$) (A' and c♯, respectively). In example 2.3b, similar procedures are applied to the perfect fourth *e*-*a* (165 Hz and 220 Hz) resulting in the frequencies of 55 Hz (the difference tone) and 385 Hz (the summation tone). As shown in these examples, the same difference tone produced by either forms of the power chord is actually below the register of a standard guitar tuning (also, see Walser 1993: 43).

The previous example is simplistic. In all musical tones, there is always more than just the fundamental harmonic sounding. Furthermore, as the guitar distortion emphasizes upper harmonics, these harmonics are even more relevant to the discussion on heavy metal chords. Thus, further discussion on the fifth *A*-*e* (or the power chord $A^5$) calls upon including the upper harmonics of both tones. The next example 2.4 shows *A* (= $f_1$) and *e* (= $f_2$) with their first eight harmonics (1*f* – 8*f*) and their frequencies. As already stated, all of these frequencies (with many others) are present in the fifth *A*-*e*. In example 2.4b, there are the harmonic frequencies of both the idealised justly intoned, and those of the equal-tempered *e*.[10]

[9] See, e.g. Lindley 2004.
[10] The equal-tempered frequencies can be found from e.g. the Main Catalogue of DPA Microphones, "How to read microphone specifications" 2004: 73. Note, that the frequency of *G#'* is erroneously reported as 56 Hz, while it should be 52 Hz.




Example 2.4. The first eight harmonics of *A* and *e*.

As can be seen in example 2.4, some of the harmonics of the justly intoned *A* and *e* match perfectly and thus intensify each other (e.g. $3f_1$ and $2f_2$, $6f_1$ and $4f_2$). This amplification happens, of course, only if the two notes are in the same phase – but if they are played together at the same time, they naturally are. For this study, we can safely make that assumption when studying chords and intervals played simultaneously on the guitar.

Furthermore, combination tones are created not only by the fundamentals but by the upper harmonics, too (see, e.g. Benade 1976: 256). These harmonics give rise to difference tones presented in example 2.5.

| Combination | Hz (Just) | Hz (Equal) | Notation equivalent |
|---|---|---|---|
| $f_2$-$f_1$ | 55 | 54,82 | *A'* |
| $2f_1$-$f_2$ | 55 | 55,18 | *A'* |
| $2f_2$-$f_1$ | 220 | 219,64 | *a* |
| $2f_2$-$2f_1$ | 110 | 109,64 | *A* |
| $3f_1$-$f_2$ | 165 | 165,18 | *e* |
| $3f_1$-$2f_2$ | 0 | 0,36 | (inaudible) |
| $3f_2$-$3f_2$ | 165 | 164,46 | *e* |
| $3f_2$-$2f_1$ | 275 | 274,46 | *c#'* |
| $3f_2$- $f_1$ | 385 | 384,46 | *g'* |
| $3f_2$-$4f_1$ | 55 | 54,46 | *A'* |
| $f_1$+$f_2$ | 275 | 274,82 | *c#'* |
| $2f_1$+$f_2$ | 385 | 384,82 | *g'* |
| $2f_1$+$2f_2$ | 550 | 549,64 | *c#''* |
| $3f_1$+$2f_2$ | 660 | 659,64 | *e''* |

Example 2.5. Some combination tones of *A* and *e*. The resulting frequencies are given in both justly tempered and equal-tempered system.

These arithmetic operations can be continued indefinitely, but a clear emphasis on the original harmonics of *A* (and their octave equivalents) can be seen even from these. The simplest combination tones – the first two of which are the most prominent (Rossing 1990: 152) – give frequencies of that equal to A in different octaves. However, the harmonics and their difference tones work exactly like this only in the idealized situation where the interval of the fifth is justly intoned. If the same calculations are made with having an equal tempered fifth as a starting point, none of the harmonics or their combinations match perfectly. If we consider guitar tuning, the real situation is somewhere in between, and dependant on the situation (chord, its positioning, overall tuning). Sometimes it is closer to the just tuning and sometimes to the equal-tempered tuning. For example, if the fifth is equal-tempered, the difference tone $2f_2$-$f_1$ (219,64 Hz) would be 0,36 Hz lower than the harmonic $2f_1$ (220 Hz). This will result in

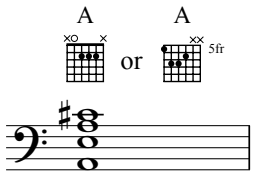
acoustic beating[11] between the two. However, the difference of these two frequencies is very small – the beating takes place only once in about every third second (i.e. 0,36 times per second) – and thus, the effect is not very noticeable (see Brian J. C. Moore 2003).

Another matter that can be read in example 2.5 is the frequent occurrence of C♯. This means that the major third of the A power chord is present even though it is not actually played. Further studies are needed in what actually sounds, and how strong the different harmonics and combination tones are when played on the guitar. However, there is some evidence that guitar players are aware and even make use of this "virtual" major third. For example, this is clear in Pete Townshend's following statement: "None of the shapes [i.e. chords] that I play with loud distortion have a 3rd, because you hear the 3rd in the distortion" (Resnicoff 1989: 80). Even though the discussion above is based on an idealised situation where these harmonics are clear, it is evident that the sound of a power chord is richer and more complicated than just the sound of a plain fifth. Townshend continues:

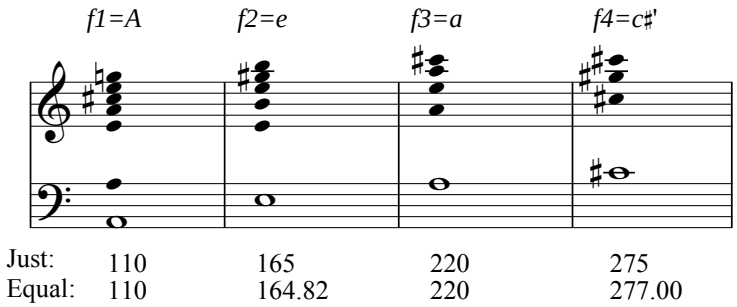
> [With a power chord] you're getting the second- and third-harmonic distortion, so the first note you're hearing is the 3rd [harmonic], the second note you're hearing is the 4th [harmonic], and the last note you're hearing is the 5th [harmonic], so if you *played* the 3rd, you're going to get a note which is 4th [harmonic] up from *that* [--]. That sound I can't stand is people playing a complete *C* chord with fuzz. They're actually getting something like a *C13*. (Resnicoff 1989: 80.)

Townshend's rejection of employing the thirds with distortion can have some explanation in the following. I start by comparing the notes of a major triad with the harmonic components of its single notes. In example 2.6 there is an A-major triad in one of its typical guitar voicing. Example 2.7 shows some of the harmonics of every individual note of example 2.6.

[11] See, e.g. Greated 2003a.

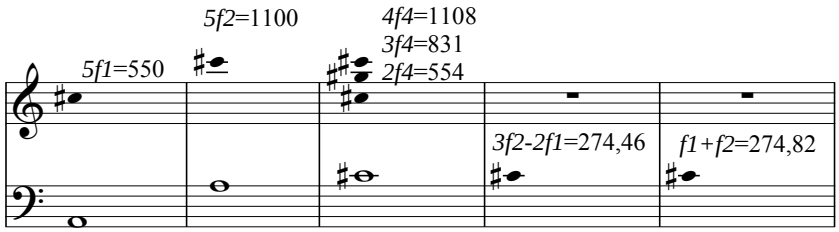



Example 2.6. A-major triad.




Example 2.7. Lowest harmonics of A-major triad's single tones. The fundamentals are with open note heads, and the upper harmonics with full.

In example 2.7 the vast number of possible combination tones between the chord tones can easily be seen. Now, all the arithmetic that was used in example 2.5 for calculating combination tones must be applied to these four fundamentals and their harmonics. Justly tempered intervals give no beating (see Appendix I). As can be seen in Appendix I, the combination tones are equal to the harmonic partials of the fundamental tones. The dissonance rises here between upper harmonics that occur in the area of $7f_1$ (770 Hz), $5f_2$, and $3f_4$ (825 Hz both), and $4f_3$ (880 Hz) – the cluster of three minor seconds that presents a dissonance for most of us. Although the justly tempered harmonics can be substantial enough to create a dissonant effect when distorted, the equal-tempered A-major triad raises much more complex sound. As shown in Appendix II, every occurrence of $f_4$ causes substantial beating with some other harmonic. For example, If only the areas of $c^{\sharp}$, $c^{\sharp'}$ and $g^{\sharp}$ are taken into an account, there are several beating frequencies (example 2.8).



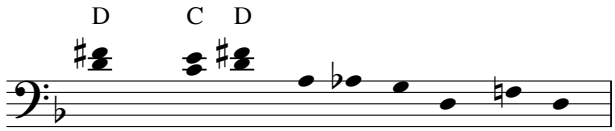

Example 2.8. Some harmonics and combination tones of equal-tempered *A*, *e*, *c#'* and *a*.

As shown in example 2.8, there are several harmonic components with different frequencies around various occurrences of C♯. For instance, the fifth harmonic of $f_1$ ($5f_1$ = 550 Hz) and the second harmonic of $f_4$ ($2f_4$ = 554 Hz) are both around $c^{\sharp\prime}$ and cause there a noticeable beating of 4 beats per second. Clashes of different harmonics and combination tones of example 2.8 can be found in the following table (example 2.9).

| Notation | Component 1 | Component 2 | Beats per second |
|---|---|---|---|
| $c^{\sharp\prime}$ | $5f_1$ = 550 Hz | $2f_4$ = 554 Hz | 4.00 |
| $c^{\sharp\prime\prime}$ | $5f_3$ = 1100 Hz (= $10f_1$) | $4f_4$ = 1108 Hz | 8.00 |
| $c^{\sharp}$ | $3f_2 - 2f_1$ = 274,46 Hz | $f_4$ = 277 Hz | 2,54 |
| $c^{\sharp}$ | $f_1 + f_2$ = 274,82 Hz | $f_4$ = 277 Hz | 2,18 |

Example 2.9. The table of some beating components in A-major triad.

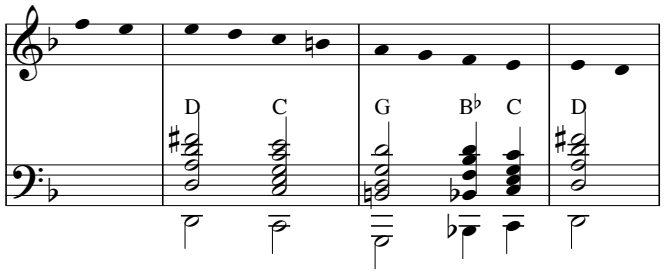
In conclusion, the C♯ of the played equal-tempered major third is not the same C♯ that is generated as a harmonic partial of the other original tones. This could be the reason for the rejection of the third by most heavy metal guitarists – the third is considered dissonant because of the beating. However, in earlier heavy metal or in preceding blues-rock the major third has been employed. For instance, the blues-derived opening riff of Cream's "Sunshine of your love" (1968) contains a sequence of altering D-major and C-major triads (with often-omitted fifths) (example 2.10).




Example 2.10. Major triads and melodic motion in riff to Cream: “Sunshine of your love.”

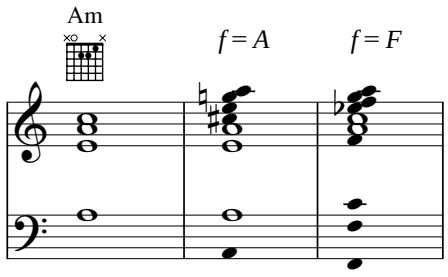
The reason could be that the distortion used by, for example, Eric Clapton of Cream in the late sixties was somewhat milder than that used by, for instance, Judas Priest in the mid eighties. If the distortion is milder, the upper harmonics are not emphasized and the combination tones are not generated in sufficient quantities to cause them to clash too much with the major third – although, aural analysis reveals some amount of “crispness” in the sound.

However, blues-rock guitarists often chose to use triads with major thirds even though those might be seemingly out of mode. This is evident in the previous example 2.10, where the overall tonality and the melodic riff are built around D-minor blues. This is even clearer in Cream’s “White room” (1968) that is clearly built on a D-minor (or, on D-Dorian). Still, the chord on the first (and every) degree is a major triad (example 2.11).




Example 2.11. “White room.”

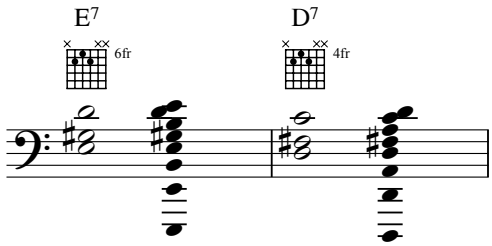
This favouring of a major instead of a minor third can be explained in a similar way as was done with the power chord. Even if the blues-rock guitarists of the late sixties had lesser degree of distortion, they had enough to create prominent harmonics around the major third. With high enough volume the distortion would catalyse the rise of a harmonic series other than that of the chord root (example 2.12).




Example 2.12. A distorted A-minor triad with its resultant harmonics. The original pitches are with open note heads.

A minor triad, when distorted, produces at least two strong harmonic series. By their harmonics and resulting combination tones, the chord root *a*, its octave doubling *a'* and the fifth *e'* emphasize the harmonic series of *A* (110 Hz). On the other hand, the third *a'-c''* gives rise to a different harmonic series – the series of *F*. (For comparison, see Tolonen 1969: 178-179.) This makes the minor triad much more dissonant than any previously mentioned structure. In fact, the occurrences of a minor third with heavily distorted guitar are very rare. In the following, these kinds of acoustically problematic chord structures will be addressed as *vertical dissonances*.

The previous discussion on the acoustics of chord structures could probably be applied to minor seventh of the dominant seventh chord structure, too. This is the structure used frequently in blues-derived rock. The structure of the dominant seventh chord is straight from the harmonic series of the fundamental – dependant on voicing, at least the fourth, fifth and the seventh harmonics are present. This is the voicing of the next example "Politician" (1968, again from Cream), presents the dominant seventh chords on bars 9-10 (example 2.13).




Example 2.13. Dominant seventh chords with implied harmonics in Cream: "Politician."

In the following chapters those chord structures, which abide by the lowest partials of the harmonic series of the fundamental tone, will be addressed as *Acoustic chords*. In the following analyses, a special symbol is used to identify them – throughout this study, a tilde (~) is placed above an analytical symbol. For instance, the D-major chord in "White room" could be labelled with Ĩ. This special labelling has been used because, as has been shown here, these kinds of chords, which seem to be out of key or mode are quite frequently used in heavy metal. Yet, they have not been receiving enough scholarly attention in music analysis.

The discussion on the acoustic features of heavy metal chords has been theoretical here. However, I believe that the ideas presented can offer a fruitful staring point for further studies.

# 3. The Problem with the Chord Roots of Dyads

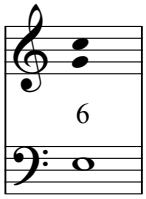
Another issue that has not been studied enough is the subject of chord roots and how they relate to the bass note (i.e. the lowest sounding note). In heavy metal they are usually the same. This apparently trivial question is important because since Jean-Philippe Rameau's *Traité de l'harmonie* (1971 [1722]) it has been a common practise to think in terms of chord inversions. However, as will be shown below, in heavy metal practises chord inversions do not play that significant a role. Furthermore, heavy metal chord constructing seems to be more similar to the intervallic practises of *figured bass*.

## 3.1. The Chord of the Open Sixth

Renaissance theorists like Gioseffo Zarlino thought in terms of intervals (Webster 2004). For example, in his *The Art of Counterpoint* (1968 [1558]) Zarlino describes how harmony is constructed of *intervals* without once referring to *chords*. In the Baroque era, Zarlino's views on harmonic foundations gained importance in the practise of *basso continuo*. In continuo playing, or writing, chords played an important role – but unlike today's common way of thinking, the chords were thought as various intervals built on the bass as designated by the figures above it (see, e.g. Williams & Ledbetter 2004). Rameau's innovation was to classify chords by their constituents in a way that gave an important role to the imaginary bass line called the *fundamental bass*.[12] Basically, the concept of fundamental bass means that detection and determining of a chord's root is made by means of "restoring" a chord to a stack of thirds (see Rameau 1971: 40). To accomplish this, the chord's constituents are rotated as much as needed. For example, the notes of a chord E-G-C would be rotated to C-E-G. This idea gained wide acceptance because it diminished the enormous amount of different chord classes to a much lesser number. Furthermore, Rameau's innovation was important to harmonic thinking because now the E-G-C belongs to the "family" of C-chords, when before Rameau it had belonged to E-chords (example 3.1). In other words, with *figured bass* of continuo the note determining the chord is the bass note,

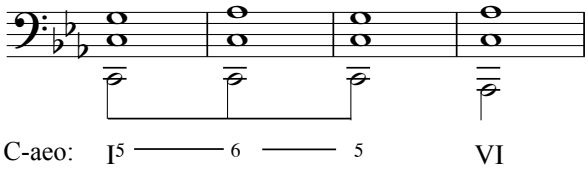
[12] See, e.g. "Fundamental bass" 2004.

whereas in Rameau's concept the chord-defining note is the lowest note of stacked thirds (see, e.g. Riemann 1977: 220).




Example 3.1. An E6-chord with figured bass and the implied harmonies on the treble clef.

In most heavy metal there is such a strong emphasis on the bass notes that the analysis should also rise from them. This is especially true with dyads. I will clarify this with the following examples presenting an open-minor-sixth dyad in two different power chord surroundings. These minor-sixth dyads often come between power chords, and (being acoustically more unstable than power chords) are used in various kinds of passing motions. Like power chords these sixths are quite rich in sound – due to their distorted sound. Hence, in this study any harmonic structure played by guitar is addressed as "chord," be it a dyad, triad, tetrad, or a more complex form. The verse of Accept's "Metal Heart" (1985) starts with the chord progression shown in example 3.2.



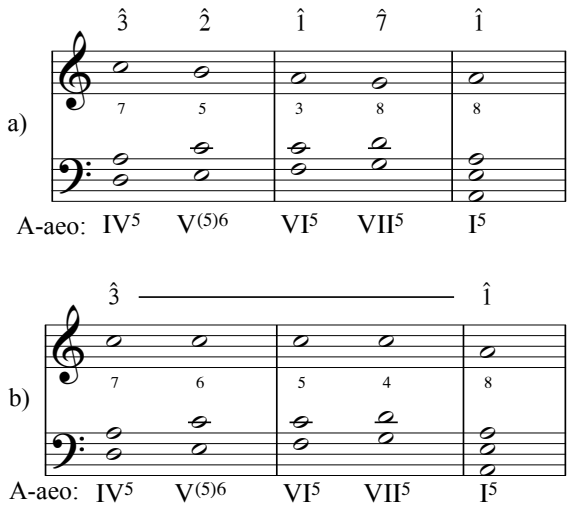

Example 3.2. Chord progression in the verse of Accept: "Metal Heart."

The guitar applies power chords and open-sixth chords in C-Aeolian mode (C-aeo). When the guitar moves from the first $C^5$ power chord to the sixth *c-a*♭, there is no change in chord root. That is to say that in the first three bars the harmony is on the first degree chord – only after that it moves to VI. This is due to the bass remaining in C and the overall heavy emphasis on C – for example, by doubling in different octaves. Only the second occurrence of *c-a*♭ coincides with the chord root moving from C to A♭ – with help from the bass. Then all the parts (the guitar and the bass) work together for

the harmonic series of A♭. In that sense, the sixth in the mm. 2 is quite dissonant. There are more differing harmonic overtone structures sounding simultaneously – naturally, that means more complexity and more beating, too.

Although this example is easy to explain as a passing note in the top voice, it is noteworthy that in this case the chord of the open sixth (in mm. 2) belongs to a different category than the major third (in mm. 4). Unlike Rameau would say, the chord C-A♭ belongs here to the same chord category as C-G – that of the I degree chords – whereas the A♭-C belongs to the VI degree chords. Actually, this is quite near to Riemann's (1961: 527-528) *Leittonwechselklang* that refers to replacing a chord's fifth with its upper, or a chord root with its lower leading tone. I suggest verifying this analysis by listening to the chord progression, and thinking at which point of time the change of chord root can be perceived.

In Judas Priest's "The Sentinel" (1984) the open minor sixth is present in a different passing motion (example 3.3). The example is of the section leading to the chorus, and is in A-Aeolian mode.



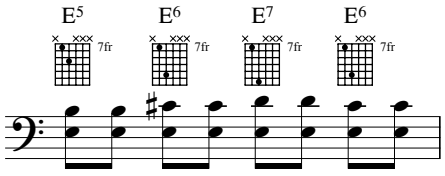

Example 3.3. The chords and the vocal melody to Judas Priest's "The Sentinel" including two occurrences of an open minor sixth. With melodic tones I use Schenkerian numbers with carets (^) to denote the relation of a tone to the tonal centre (see, e.g. Schenker 1979: 13). The numbers between the staves denote the interval between the bass and the melody – according to the customs of traditional counterpoint (see Mann 1971: 29).

According to Rameau's "stacked thirds" theory, the second chord (of the open sixth) should be analyzed as a III degree triad in its first inversion. However, this proves to be problematic. The first and the most important line that strikes the ear is the lowest: D-E-F-G-A. In this case, too, the bass line is doubled several times in several octaves, leaving the second chord's sixth with little influence on chord-root detection. The sixth is rather a more dissonant variation of the more common fifth. Moreover, the vocal melodic line in example 3.3a is on $\hat{2}$ forming the fifth with the base and leaving the sixth even more isolated. Example 3.3b offers an interesting variation. Now, the vocal melody is with the sixth and remains on $\hat{3}$ until forming a blues-like melodic cadence $\hat{3}$-$\hat{1}$ to the tonic (cf. Martin 1996: 28). Still, the C does not gain enough strength to act as a chord root. In light of this example, it seems that the melodic line has two possibilities concerning the sixth. It can be ignored by following the harmonics of the fundamental (as in example 3.3a), thus the sixth is left as a colouring dissonance, or, its harmonic colour can be emphasized as was shown in example 3.3b.

If a larger scale analysis is applied to "The Sentinel," the sixth's note C would be a preparation of the next chord's fifth. Still, the open-sixth chord is common enough in heavy metal to deserve a symbol of its own. The symbol $V^{(5)6}$ I have chosen to use here calls for an explanation. The chord symbols used in guitar magazines and commercial note books have typically two ways of handling the minor sixth. It is either marked as an inversion (the sixth in "The Sentinel" would be C/E), or as an augmented fifth ($E^{+5(\text{no 3rd})}$). For the reasons discussed above, a symbol implicating an inversion is out of the question, and calling a minor sixth an "augmented fifth" is modally and tonally unacceptable. Moreover, having discussed the subject with several heavy metal guitarists, many spoke specifically of "the chord of the minor sixth" when referring to these kinds of chords. Thus, the first natural choice would have been a single number "6." In the analyses made here, the numbers are interpreted according to the mode in use. Very similar to figured bass numbering, if we had an A-minor mode, the number 6 on a bass note E would give us an E and a C (and a G in figured bass). This single 6 would have made a suitable parallel to power chords label – 5 for "fifth", 6 for "sixth." Indeed, they seem to be parallel, at least where the guitar's instrumental techniques are in question – it is very easy to change a fifth into a minor sixth by moving one finger only. However, the number six carries at least two strong connotations. In traditional

harmony analysis (e.g. Piston 1962: 44) it stands for "first inversion" – for instance, $I^6$ is the first degree triad with its third on the bass. In chord symbols used in jazz and pop/rock it commonly means a triad with an added, always a major sixth (e.g. $C^6$ includes notes C-E-G-A). The label chosen here denotes the chord root (i.e. the bass note) with the Roman numeral, and the fifth's absence is denoted with parentheses. However, since the bass note is usually strong, its third harmonic partial (i.e. the interval of the fifth) should be relatively strong, too. In a way this might be even more descriptive symbol than a single number 6, because $V^{(5)6}$ can be read as taking the fifth's presence in harmonics into account.

The following example will give more grounds for intervallic thinking among rock musicians. The basic rock 'n' roll accompaniment pattern employed by Chuck Berry and many others includes the intervals of the fifth, major sixth and minor seventh that follow each others in a way presented in example 3.4. For instance, the beginning of Steppenwolf's "Born to Be Wild" is based on this pattern. The chord symbols for this example follow a *Guitar World* magazine's "performance & analysis" column by Matt Scharfglass (2004: 111). Note that these chord symbols are used to denote single intervals, not triads or tetrads.

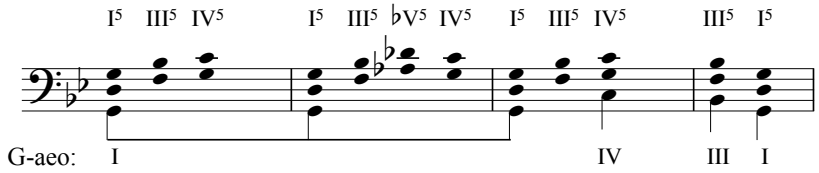



Example 3.4. An intervallic rock 'n' roll guitar pattern.

## 3.2. The Chord of the Fourth

"The 4th has a unique position in Western music because it has been regarded as a perfect interval (like the unison, 5th and octave) and a dissonance at the same time" (Drabkin 2003). The chord of the fourth has at least these two uses in heavy metal, too. Sometimes the fourth's upper note is perceived as a chord root and sometimes the lower. When the upper tone is perceived as the chord root, it works as an inverted fifth

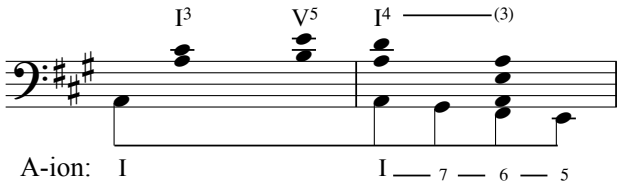
power chord. In that case, the harmonic series of the upper tone is an overriding element in perception. This is the case of the guitar riff in Deep Purple's "Smoke on the Water" (1972). Deep Purple's guitar player Ritchie Blackmore often applies these kinds of power chords particularly in riff-type guitar patterns. In the cases like this the bass usually works as a pedal point (example 3.5).




Example 3.5. The riff to "Smoke on the water." This reduction shows the guitar and the bass parts.

Actually, the guitar's power chords can be heard as a single melodic line, the sound of which is strengthened with a perfect fourth below. The pedal point in the bass establishes the tonal centre effectively and joins only the last four power chords.

However, sometimes the tonal environment and voice leading practises call for another kind of interpretation. When the perfect fourth is in relation to the bass (i.e. the fourth's lower note is on the bass), it is usually considered as a "dissonance" (Salmenhaara 1968: 25) or at least "unstable" (Drabkin 2003). In any case, it has to resolve to a third or a fifth. This kind of usage of the perfect fourth can be found, for example, in the songs of Judas Priest. For instance, "Devil's Child" (1982) has an E-Dorian $I^5$-$IV^{4-3}$-$I^5$. This suspended fourth is also in Ozzy Osbourne's "Crazy Train" (1981) (example 3.6).




Example 3.6. Reduced riff to the verse of Ozzy Osbourne: "Crazy Train" with the guitar and the bass parts included.

The guitar's dyads are built on the bass' pedal point on the I degree. As it is marked in the analysis, the bass is embellished with passing notes (8)-7-6-5. The perfect fourth's

resolution on the first degree sounds just like a classic suspension-resolution $I^{4-3}$. However, the most intriguing feature in this example is that the third of the last chord (i.e. the resolution tone) is not actually played – in the analysis it is therefore put in parenthesis: (3). Still, it can be heard quite clearly. There are two main reasons for this, the first being tied to the tonal expectations. The ear of the average Western listener is so well trained in major/minor tonal music that they expect to hear the fourth going to the third. The fact that this kind of fourth on the I degree chord moves to a more stable chord on the same degree is enough to create expectations for hearing a C♯. Actually, I have heard many guitarists playing out the originally missing C♯ – thus, it appears that the C♯ is relevant to them. Moreover, the C♯ is acoustically present in the chord's harmonics – as presented in examples 2.4 and 2.5 of the previous chapter. Also, note Pete Townshend's statement cited above: "I hear the third in distortion."

Thus, the perception of this musical phenomenon is affected by both culturally-based expectations and the chord's acoustic structure. It seems that the role of tonal expectations in the perception of the fourth are enforced when we have a minor-type mode – as in "Smoke on the Water" (example 3.5 above). In "Smoke on the Water" the mode is a minor-related Aeolian, whereas "Crazy Train" is in Ionian (i.e. common major mode). The fourth in the bass does not inflict the need for resolution in "Smoke on the Water" – at least not that strong. On the contrary, it is quite difficult to hear a 4-3 resolution between the $IV^5$ and the $I^5$. In that case, the modal/tonal context (minor third in the mode) and chord structure's acoustics (major third as a harmonic partial) are conflicted. My ear decides for the mode here. Moreover, arithmetically the major third seems to be weaker in the interval of the fourth than in the fifth.

Interpreting "Crazy Train's" fourth as a suspension is further supported by the fact that the "fourth against the bass" is on the strong beat and its resolution on the weak, whereas in "Smoke" the situation is reversed: the fourth against the bass ($IV^5$) is on a weak beat and the following $I^5$ is on the strong. Furthermore, the role of the bass is different in these two riffs. In "Crazy Train" the bass acts as a pedal point only for the first bar's $V^5$. It is really not a proper pedal point at all – the chords $V^5$ and $I^{4-3}$ are likely to be perceived as passing motions against the bass. On the contrary, the pedal

point that carries almost throughout the whole riff in “Smoke on the Water” is much stronger – it is long enough and has more changing dissonant motion above it.

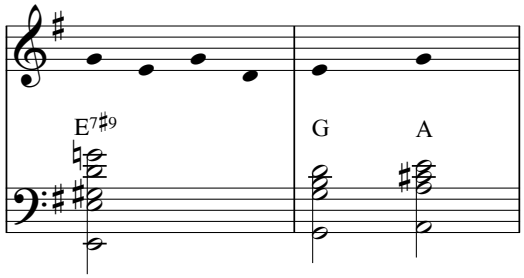
In using the fourth there is still one point to make – a point that is pertinent to voice leading practises. The “upper note as a root” interpretation of “Smoke on the Water” is enforced by the fact that the interval harmonizing the riff’s melody does not change. Those kinds of unchanging intervals tend to melt into the main line, thus becoming a part of its sound. In those cases the melody and its harmonizing interval are likely to be perceived as a whole, or, as a one-note melody. This is not the case with diatonic intervals that act more under the traditional voice leading, and have more independence. For example, the harmonizing of “Crazy Train’s” riff with various diatonic intervals creates a need for resolutions within the riff more than the harmonizing with chromatic parallel intervals would.

# 4. Scales, Modes and Voice Leading in Heavy Metal

In the traditional Western art music, at least since the early eighteenth century (see Riemann 1977: 162-163), there have been only two modes – the major and the minor (see, e.g. Harrison 1994: 17; Salmenhaara 1980: 197). However, the scales, or modes, used in heavy metal vary much more and can be traced to at least three primary sources – 1) the blues-derived pentatonicism, 2) the so-called church modes and, 3) the Western major/minor tonal influences.

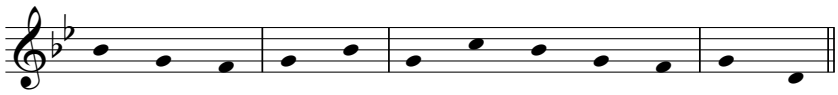
## 4.1. Pentatonic and Blues Scales

As heavy metal is a subgenre of rock, its roots are in rock 'n' roll. Rock 'n' roll has its origins in rhythm & blues, and rhythm & blues in the blues (see, e.g. Wagner: 2003: 353). "[--] heavy metal guitarists who did not study the blues directly learned secondhand, from the cover versions of Eric Clapton and Jimmy Page or the most conspicuous link between heavy metal and blues and r&b, Jimi Hendrix" (Walser 1993: 58). Hence, it follows that in heavy metal there are elements that originate from the blues. One such a feature is the use of pentatonic scales. The blues-derived pentatonicism is an important feature especially in early heavy metal, but some of its influence has remained in later metal, too. As an early example, the vocal melody in Jimi Hendrix's "Purple Haze" (1967) is built on E-minor pentatonic scale degrees, as are the chord roots (I-III-IV), too (example 4.1).



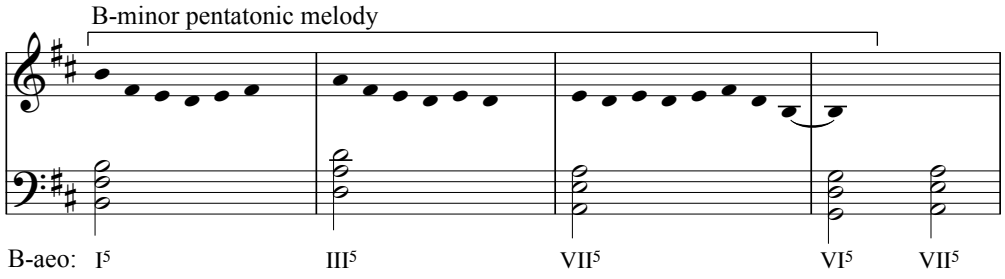

Example 4.1. "Purple Haze."

Another early example is the vocal melody in Deep Purple's "Speed King" (1970) that is basically plain G-minor pentatonic (example 4.2).



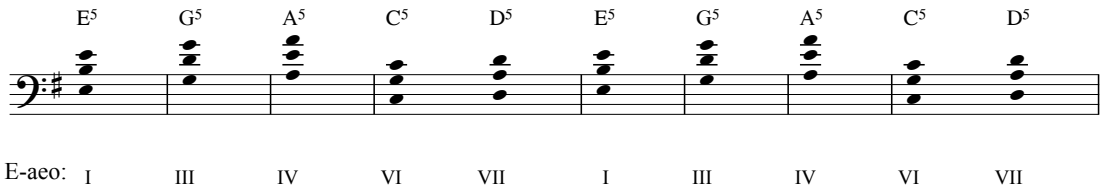
Example 4.2. Overall melodic line in the verse of "Speed King."

Pentatonic vocal melodies have been an important part of Deep Purple's later music, too. In "Knocking at Your Backdoor" (1984) the vocal melody is clearly pentatonic. However, with the chords the overall melodic-harmonic environment is more near to a modal Aeolian scale (example 4.3). This is one example of how pentatonic melodies can occur with harmonic environments that are derived from other sources (as will be discussed in later parts of this chapter).




Example 4.3. Overall melody and harmony in the first four bars of the verse of "Knocking at Your Backdoor."

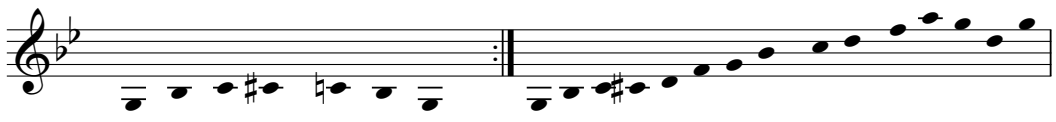
Pentatonic influences can also be detected in the choices of chord degrees, and especially their roots. The chords in "Knocking at Your Backdoor" are quite near to the following progression, the variations of which have been widely used in heavy metal from the seventies (e.g. Deep Purple: "Highway Star" [1972]) through the eighties (e.g. Iron Maiden: "2 Minutes to Midnight" [1984]). The next example is from Iron Maiden's "Running Free" (1980) (example 4.4).




Example 4.4. Chord progression of the ending section of Iron Maiden's "Running Free."

Actually, the chords in this progression follow E-Aeolian mode (with an absent second degree), but note that the chord roots can be seen as constituents of a pentatonic scale – although a different pentatonic scale than in the previous examples. This scale is the third mode of C-major pentatonic scale – i.e. it starts from the third note of C-major pentatonic (see, e.g. Backlund 1983: 42). The popularity of the scale degrees following this scale could be due to the conscious (or not so conscious) efforts to avoid scale degrees, which as triads would form minor chords. For example, Aeolian V degree would be a minor.

In addition to different pentatonic scales, there is one particularly important scale used in heavy metal riffs and solos – the so-called blues scale that is a minor pentatonic with an addition of the tritone (i.e. augmented fourth / diminished fifth) (e.g. Burbat 1988: 115). Actually, there are several different scales known as the "blues scale," but this one seems to be very specific to heavy metal.[13] Next example shows the main riff's melodic line in Black Sabbath's "Rat Salad" from *Paranoid* (1970) (example 4.5). However, this is not a pure blues scale riff – at the end there is an A on the second degree. In fact, heavy metal music's riffs, melodies, or harmonies rarely follow strictly only one scale.



Example 4.5. Melodic line of the main riff in "Rat Salad."

As the next chapters show, there are other musical characteristics in heavy metal that are at least as important as those derived from the pentatonicism. While putting e.g.

[13] Also, see *Iron Maiden Guitar Tab Edition* 1994: 6.

blues, jazz and rock under a single label "Afro-American music," John Shepherd (1991: 129, 131) states that "many Afro-American melodic lines seem to be pentatonic." While this might be true in general, it is certainly a simplistic view regarding heavy metal.

## 4.2. Modal Scales and the Modal System

If we put aside those readings of the term "mode" that are applied to rhythms and intervals, the term "has always been used to designate classes of melodies, and since the 20th century to designate certain kinds of norm or model for composition or improvisation as well" (see, "Mode" 2003). Despite that "mode" is a concept in the history and theory of European music it is also a modern musicological concept that has been applied to non-Western music since the late 18th century (Powers & Wiering 2004a, Powers & Widdness 2004; also, see Pennanen 1999: 42-44). The modal concept has been applied to the study of Anglo-American folksongs, too (see, e.g. Powers & Cowdery 2004). In addition, the composers of the Western major/minor tonal era (e.g. Beethoven, Schumann, Chopin, Liszt, Bartok, and Debussy) have occasionally used the modal scales of earlier periods – derived from various folk music traditions or from the old church modes (see, e.g. Porter 2004).

With no doubt Anglo-American folk music traditions have been significant in the formation of other popular styles such as rock. This influence can be traced back at least to the 17th or 18th centuries (see, e.g. van der Merwe 1989: 16-17), but I restrict the following to more recent influences. Some modal qualities of heavy metal come directly from the folk music of the 1960's. For example, Jimi Hendrix's "All Along the Watchtower" (1983 [1968]) is originally written and performed by Bob Dylan (*John Wesley Harding* 1967), and Judas Priest made a cover version of Joan Baez's "Diamonds and Rust" (*Diamonds & Rust* 1975) for their album *Sin After Sin* (1977). Both Dylan and Baez derived much of their music from the traditional Anglo-American folksong. Furthermore, both of these songs have melodic and harmonic qualities that can be described easily as "modal" (see the analysis of "All Along the Watchtower" below).

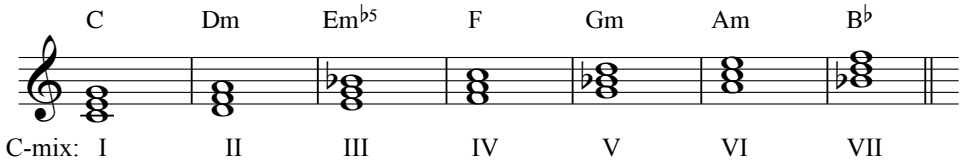
If we can speak about folksongs as "modal" (as in Powers & Cowdery 2004) we can also apply the same term to heavy metal. Scales resembling the church modes had been used by e.g. The Beatles, but since the 1970's many guitar players have definitively become familiar with modal scales (i.e. church modes) through various music theory books (see, e.g. Walser 1993: 65-66). Actually, it seems that the melodies and harmonies of heavy metal are more often built according the modal scales than, for example, the major/minor system of Western art music. This can be seen, for example, in the importance of the chord progressions that move in thirds and seconds, and in the extended usage of the "flattened" seventh degree (see, e.g. Moore 1995). Also, the musicians have explicitly stated using modal scales as a compositional or improvisational starting point (e.g. Whitehill 1989). These facts support the use of a modal system in the analysis of heavy metal.

The modal system used here has been introduced by Allan Moore (1992; 1995; 2001). In this system the chord degrees relate to a modal scale, not to the traditional major or minor scales. In this study the terms "mode" and "modal" are used to designate scale types and not "melodic motifs" or "patterns" as it is sometimes done (see, e.g. Powers & Cowdery 2004). The names used here for the modes are those originally derived from the theories of the Ancient Greeks for the use of Gregorian plainchant and Renaissance polyphony. The scale types are mostly the same that were introduced in the 16$^{th}$ century by e.g. Heinrich Glarean (1488-1563) and Gioseffo Zarlino (1517-1590) (Powers & Wiering 2004b). Thus, here the modes refer to seven note scales (and here, to their equal-tempered equivalents), where the tones (t) and semitones (s) are distributed as follows:

Lydian (abbr. lyd): t – t – t – s – t – t – s,
Ionian (ion): t – t – s – t – t – t – s,
Mixolydian (mix): t – t – s – t – t – s – t,
Dorian (dor) : t – s – t – t – t – s – t,
Aeolian (aeo) : t – s – t – t – s – t – t,
Phrygian (phr): s – t – t – t – s – t – t
Locrian (loc) : s – t – t – s – t – t – t.

The first six of these were used in the music of the medieval and Renaissance eras. Their names also originate from those times. The difference between modern usage and the medieval is that here the modes can be transposed to any tone, whereas medieval modes were usually tied to certain notes (e.g. Aeolian is an A-mode, Phrygian is an E-mode etc.; see, e.g. Jeppesen 1975). The seventh mode, Locrian, was acknowledged by the Renaissance theorists, too (e.g. Zarlino 1968: 43). However, at that time it was basically nonexistent in practise because of the diminished fifth between its root and the fifth. However, "the alternation between E in the bass and the B♭ power chord seems unusually common in trash metal. In Renaissance and baroque music, this relationship connoted the works of the devil [--]" (Moore 2001: 179n21). This interval addressed as the "Devil in music" back then has been widely used at least since the late 1980's speed metal (e.g. Metallica's *And Justice for All…*), but already in the early 1970's by Black Sabbath (e.g. the albums *Black Sabbath*, and *Paranoid*, both 1970).

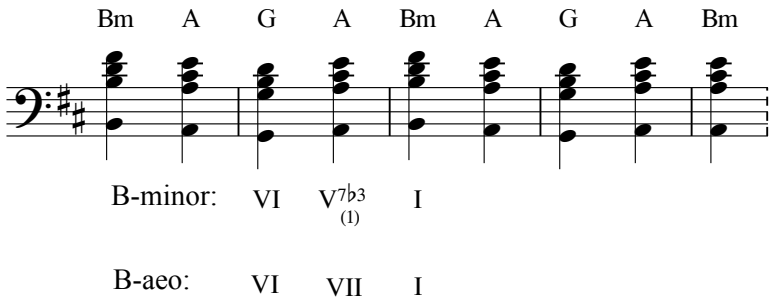
In the modal system the chords are built of the modal scale-steps, just like in traditional chord constructing. For instance, the next example shows the chords that are constructed of C-Mixolydian scale-steps (example 4.6). The symbols that denote the "key" in the beginning of the analysis are mine (e.g. "C-mix" means that the following chords are analyzed in C-Mixolydian mode).




Example 4.6. The triads of C-Mixolydian mode.

The basic advantage of Moore's system is that it makes harmony analysis much simpler and diminishes the need for chromatic symbols for "root altered" harmonies. This is especially important if we do not want to treat those chord degrees that differ from those of the traditional major/minor tonal system as some sort of deviations (this has been discussed in length by Moore 1992). Using an analytical method that has the major/minor tonal music as a starting point can lead us to see the major/minor tonal features as "normal" or "natural," and anything that differs from those features as

"deviant" (see, e.g. Everett's [2000] Schenkerian analyses of rock music). For instance, in much rock music the Mixolydian seventh degree is far more prevalent than the Ionian (i.e. major) seventh degree (see Moore 1995). The analytic labelling according to a mode makes the analysis simpler. For example, if we analyze the popular Aeolian chord progression VI-VII-I with the principles of traditional harmony analysis, the results look quite strange and hardly do justice to the music. The next example shows Jimi Hendrix's "All Along the Watchtower" analyzed with Erkki Salmenhaara's (1968: 61) principles and with the modal system (example 4.7). According to Salmenhaara, and Walter Piston (1962: 33) the analysis is made in harmonic minor. In the light of this example it is evident that the modal system makes the analysis more compact.




Example 4.7. The chord progression to Jimi Hendrix's "All Along the Watchtower" analyzed according to the traditional harmony analysis and modal system.

Note, that for Salmenhaara the A-major chord is an altered chord, thus the additional figures and chromatic markings. On the other hand if the song is analyzed in Aeolian mode there are no altered chords. The chord progression in question is in fact so common that is hard *not* to find it in the music analyzed here – for instance, it can be found in almost every Iron Maiden song (see, Iron Maiden 1980, 1981, 1983, 1984, and 1986).

While Moore's theory is sound it could have been taken further. In practise heavy metal chords do not follow one single mode, and thus, we still have to use chromatic altering to achieve the correct structure. For instance, Moore (2001: 55) analyzes Jimi Hendrix's "Hey Joe" as an Aeolian VI-III-VII-IV♯3-I♯3. The major thirds that do not abide by the mode have to be written as an altered harmony. Apparently Moore has not

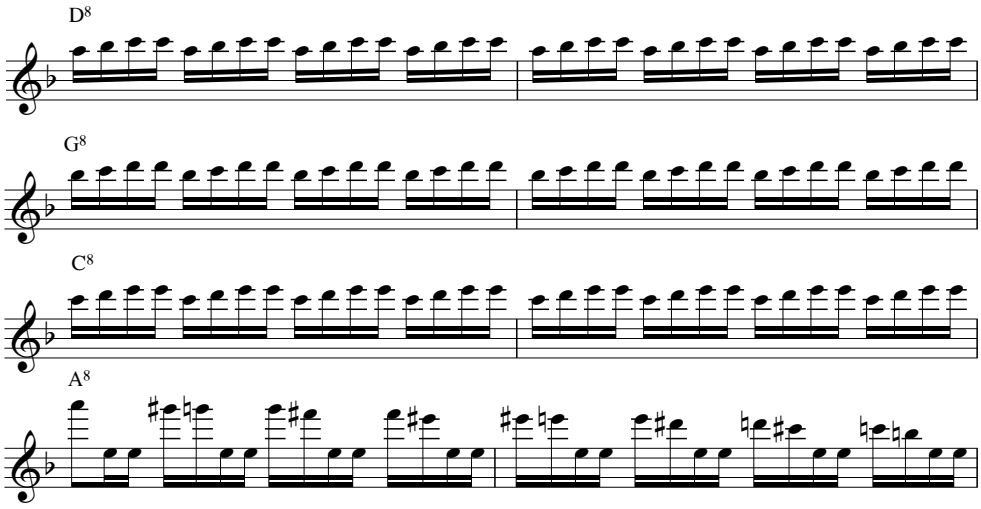
taken in to an account one important feature of rock – that the vertical structure of a chord is often more important than whether a chord abides by a certain mode. This will be discussed in detail in Chapter 5.

A further note regarding "Purple Haze" must be made here. In my opinion, it is as relevant to analyze it either as using a pentatonic or as using a Dorian mode. The Dorian interpretation could face criticism on the grounds that all the notes of the Dorian are not present. However, the problem is similar with the pentatonic interpretation, too. To be accurate, the melody that contains only of three notes D-E-G is rather a *triatonic* than a pentatonic one. If the chord roots are taken into an account, we would have a four-note scale D-E-G-A. Furthermore, if the thirds of the G-major and A-major are added, the result is close to E-Dorian – with only the F♯ missing. Both of these interpretations, whether Dorian or pentatonic, include notes by implication i.e. "filling in the missing notes" (see Pennanen 1999: 40-43). At least for the purpose this study, however, the advantages of the modal system outweigh its problems.

## 4.3. Major/Minor Tonal Influences in Heavy Metal

Although, the modal scales seem to be particularly important for heavy metal harmony, the influences of classical major/minor tonal music are a prominent part of heavy metal vocabulary. Heavy metal has taken influences from classical music at least since the early seventies. An easy way of detecting this is to note some quotations of classical repertoire in heavy metal context. For example, Deep Purple's organ player Jon Lord quotes Beethoven's *Für Elise* in the song "Speed King" (1970). Lord was classically trained, and even composed a lengthy piece called *A Concerto for Group and Orchestra* (1970) incorporating a full symphonic orchestra into the band. In fact, some other members of Deep Purple also had at least some classical training. The guitarist Ritchie Blackmore took some lessons in classical guitar before Deep Purple, and took up the cello in the late seventies (Webb 1984: 54; Rosen 1984: 59). Blackmore frequently uses melodic figures derived from Baroque composers in his guitar solos and riffs – this has been discussed in length by Robert Walser (1993: 57-107; 1992), so I will restrict myself to a single example, "Highway Star" from *Machinehead* (1972).

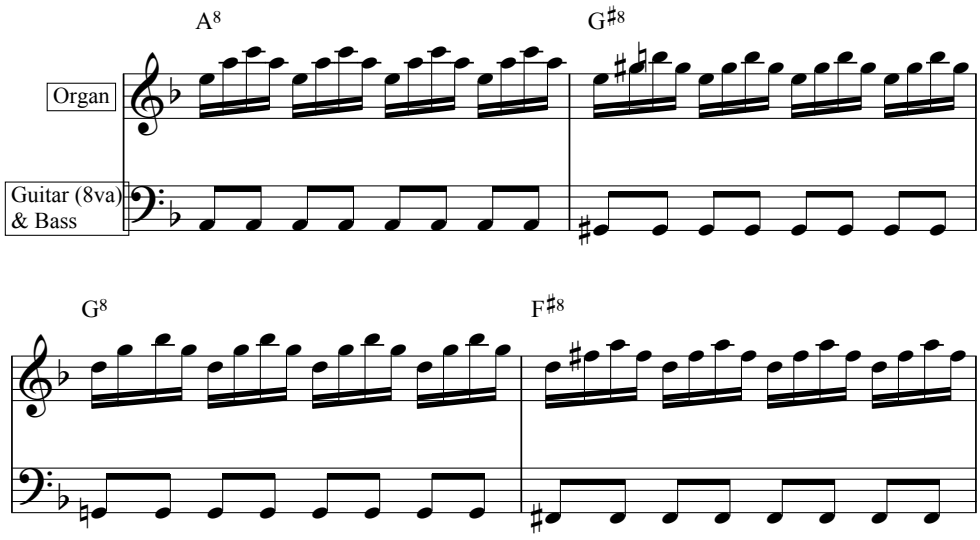
Blackmore comments the solo of "Highway Star," "it's just arpeggios based on Bach" (Rosen 1984: 62). Indeed, the guitar solo resembles the characteristic Baroque style: the fast and repeated sixteenth note pattern on the chord progression following the circle of fifths (example 4.8).[14]




Example 4.8. Excerpt from the guitar solo of "Highway Star." Only the first guitar track is included in this transcription. The chord symbols with the figure "8" denote that the accompaniment includes only the single note (possibly) doubled in octaves.

In Jon Lord's organ solo for the same song, these arpeggios have similar kinds of influences, now with a chromatically descending bass line that also was favoured by Baroque composers (see, e.g. de la Motte 1987: 74). Example 4.9 is from Deep Purple's live album *Made in Japan* (1972).

[14] Walser (1993: 65) has the same example with the guitar harmonized in parallel thirds – a note must be made here that in Walser's transcription there seems to be some mistakes on the descending chromatic line of the last two bars.



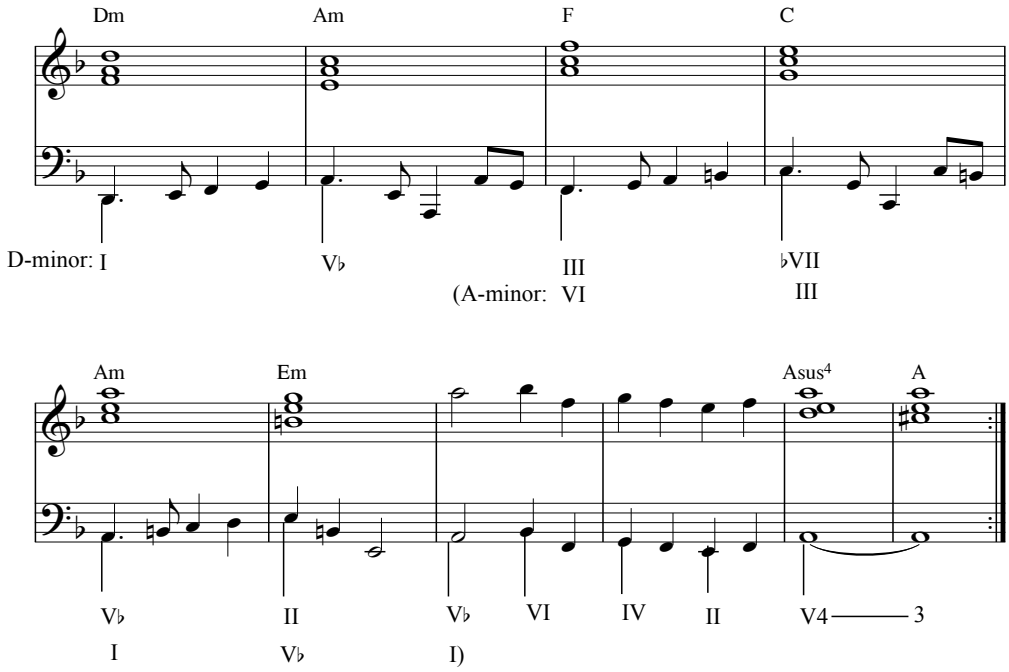

Example 4.9. Excerpt of Jon Lord's organ solo for "Highway Star."

In fact, many heavy metal musicians of the seventies and eighties were either trained in classical music or self-learned in the ways of traditional theory of harmonies and melodies.

> Throughout the 1970s, guitarists continued their experimentation with fusions of rock and classical music. Just as jazz musicians had done in the late 1940s, some rock guitarists turned to classical music theory for new musical resources. (Walser 1993: 65.)

This influence is evident in the works of musicians of the 1980's – the classical influences are not anymore quotations or borrowed patterns, rather the classical vocabulary is incorporated in heavy metal style. Guitar players such as Randy Rhoads and Yngwie Malmsten made the classical influenced repertoire a prominent part of heavy metal vocabulary (see, e.g. Walser 1993: 78-102). They had both studied classical music from an early age (ibid. 78, 93), and when turning to heavy metal brought the major/minor tonal practises with them. It is safe to conclude, that they both were well-accomplished in classical music theory. One example of this can be seen in Rhoads's notational sketches on the album cover of Ozzy Osbourne's *Randy Rhoads Tribute* (1987). The sketches include seventh chord arpeggios for all seven modes in the key of C♯-major, including their primary chords (IIm7, V7 and Imaj7 – as they typically are in jazz). Furthermore, the chord substitutions seem to follow quite normative major/minor tonal conventions (e.g. I can be substituted with VI).

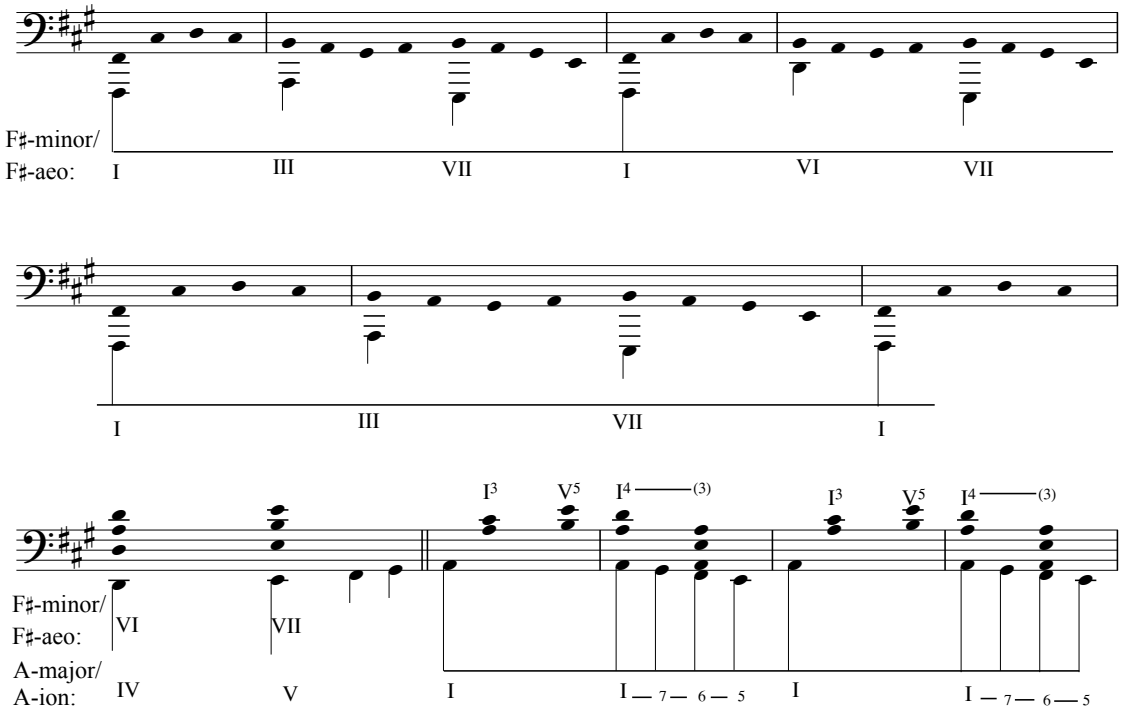
In Ozzy Osbourne's "Mr. Crowley" (composed by Rhoads) there is a very Baroque-like sequence technique used for the introduction (see Walser 1993: 79). Furthermore, the intro is played on the synthesizer using a sound resembling a pipe organ. Note that the chord progression and the bass are very much like that of the traditional *basso continuo* – for example, the bass fills in the gaps between the chord roots with stepwise passing movements (example 4.10).




Example 4.10. The intro from "Mr. Crowley."

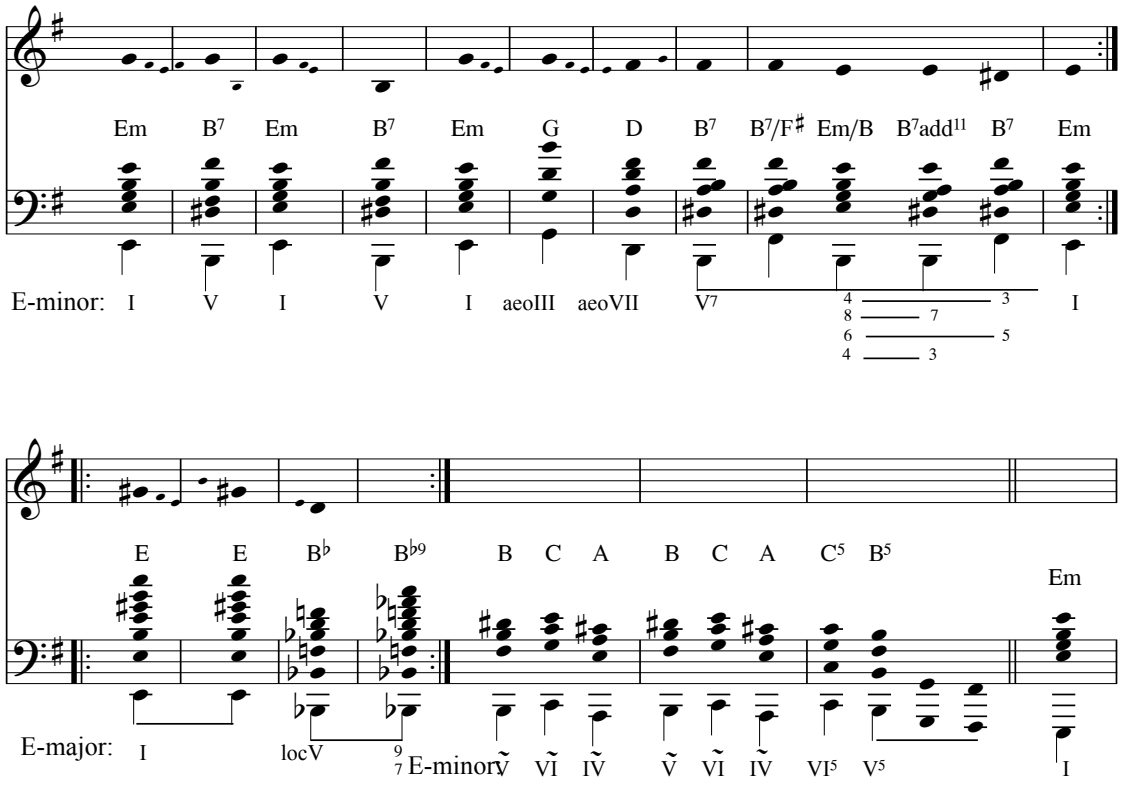
For solo parts of "Mr. Crowley" the chord progression of the intro is modified into a full circle of fifths in D-minor. "Until classically influenced heavy metal, such cyclical progressions were unusual in rock music, which had been fundamentally blues-based" (Walser 1993: 80). In another Ozzy Osbourne song, "Crazy Train" (1981), Rhoads uses yet another clearly classically influenced compositional solution. The key changes between the introduction, verse and the chorus make use of the relative minor and major keys (example 4.11; the song is analyzed in detail in Lilja 2002). The introduction (mm. 1-7) is grounded on F♯-minor, or better, F♯-Aeolian because of the usage of the extremely common Aeolian cadence VI-VII-I. On the other hand, the verse (starting in mm. 9) is in A-major (e.g. A-Ionian). The cadence in mm. 8 acts as a

transition between the two keys – it is first interpreted as an Aeolian cadence as before, but this interpretation is altered as soon as the A-major riff starts. After that, the two power chords of mm. 8 are perceived with the riff's first chord as a normative major/minor tonal cadence IV-V-I.




Example 4.11. Ozzy Osbourne: "Crazy Train," the guitar and the bass parts for introduction and the beginning of the verse.

In "Revelation (Mother Earth)" (1981) Rhoads makes use of a parallel major and minor (example 4.12). Also, the instrumentation is classically influenced – there are at least acoustic guitars (both metal and nylon string), flute, string quartet, and tubular bells.




Example 4.12. Harmonies for "Revelation." Vocal part is on the upper stave.

The verses of the song (mm. 1-10) are played with the soft sounds of acoustic guitars, a string quartet, and the flute. The overall mode is classically influenced major/minor tonal harmonic minor in the key of E – with two modal degrees from E-Aeolian. The cadence in mm. 9-10 includes interesting suspensions. For example, there are two suspended fourths that resolve separately. From mm. 11 the instrumentation changes to the distorted electric guitars. In addition, the key is changed – although there is a Locrian V degree creating tonal suspension, the overall feel of the key is E-major. This change of key is actually quite dramatic. It is noteworthy that when the guitar changes to distortion, the chord structures in use change accordingly – to the Acoustic chords discussed in the previous chapter. There are no minor triads until the return to the acoustic-based ensemble in mm. 18.

In Rhoads's own words:

"Revelation" and "Mr. Crowley" are my favourite cuts on the first LP because both of them have a heavy classical influence. I think the relationship between heavy metal and classical music is great. (Obrecht 1984: 176.)

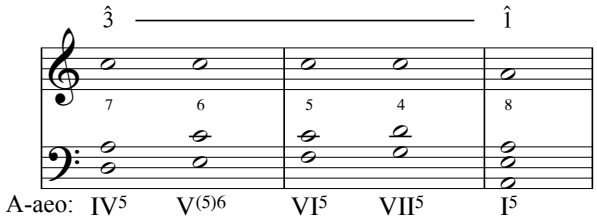
Rhoads was not the only or the first one to make heavy use of classical music, but certainly he was one of the most influential players/composers in that field for future generations:

> Rhoads's interest in music theory was symptomatic of the increasing classical influence on heavy metal, but his success also helped promote classical study among metal guitarists. Winner of Guitar Player's Best New Talent award in 1981, Rhoads brought to heavy metal guitar a new level of discipline and consistency, derived from classical models. [--] Rhoads accomplishments also contributed to the growing tendency among guitarists to regard their virtuosic solos in terms of a division of labor long accepted in classical music, as opportunities for thoughtful composition and skilful execution rather than spontaneous improvisation. (Walser 1993: 84.)

In fact, numerous other heavy metal guitarists have expressed their appropriations to classical music – starting from Jimi Hendrix (Burks 1984: 22), Jimmy Page of Led Zeppelin (Rosen 1977a: 44, 50), and John Paul Jones, the bass and keyboard player of the same band (Rosen 1977b: 104), to K.K. Downing and Glenn Tipton, the guitarists of Judas Priest (Varney 1984: 132).

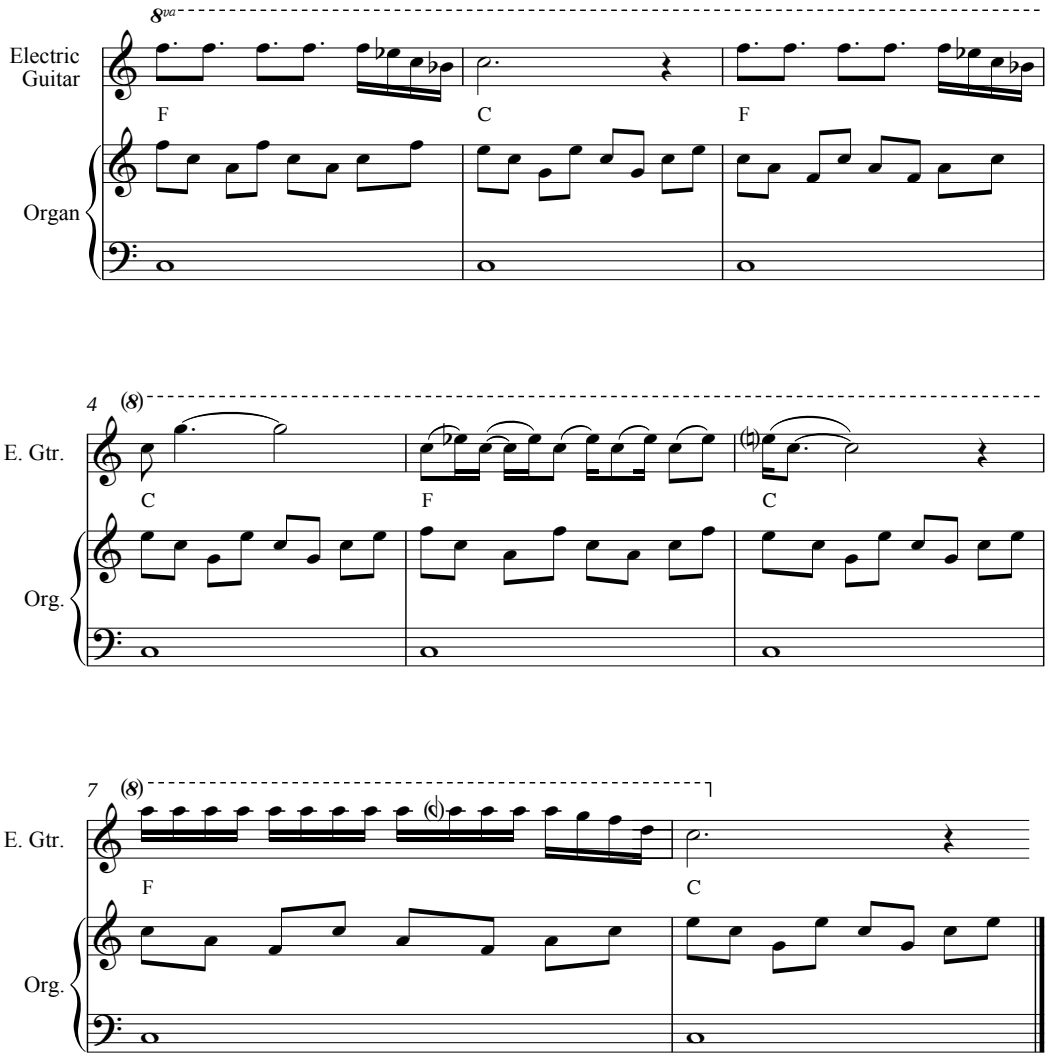
## 4.4. Mixed Scalar Contents and Voice leading Derived from Different Traditions

Often in heavy metal the major/minor tonal, the modal, and the pentatonic features act in parallel. For example, as briefly mentioned in the previous chapter, Judas Priest's modally constructed harmony in "The Sentinel" includes a pentatonic-like melodic motion from the minor third degree to the first – such motions are quite dogmatic for blues melodies (see, Titon 1977: 155). However, as Moore (2001: 150) states, in the case of heavy metal there is "a strong preference for [equal] tempered pitch" (example 4.13).




Example 4.13. "The Sentinel."

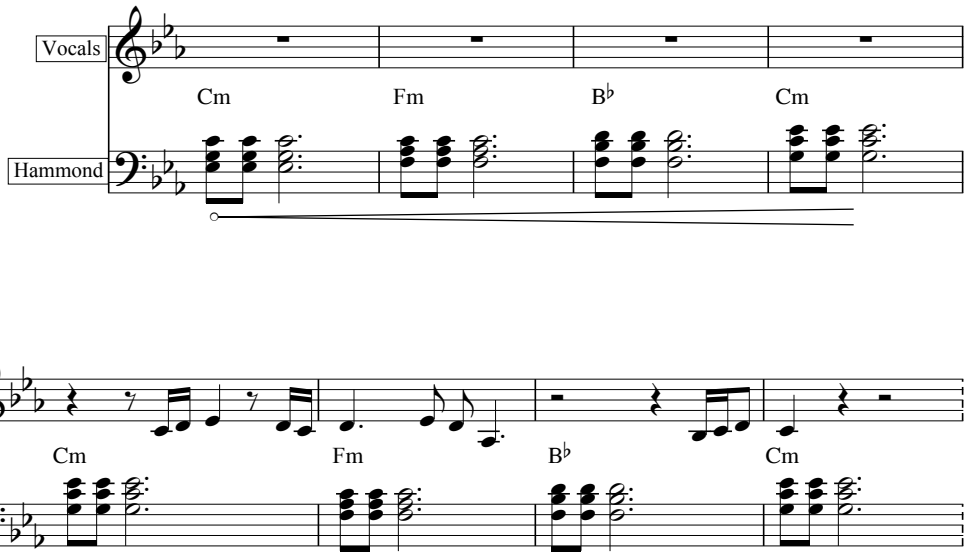
Furthermore, the interplay of different traditions is evident in the introduction of Uriah Heep's "July Morning" (example 4.14).




Example 4.14. Introduction to Uriah Heep's "July Morning."

The Hammond organ has established the key of C-major some eight bars before with the triads of IV and I. At the beginning of this excerpt, the guitar employs a C-minor pentatonic scale for the first five bars of the solo, only joining the organ in the occurrence of a major third of the C-major key in mm. 6. From mm. 7 on, the guitar part can be interpreted as being in the fifth mode of the major pentatonic – the so-called natural pentatonic mode (see, e.g. Burbat 1988: 114).

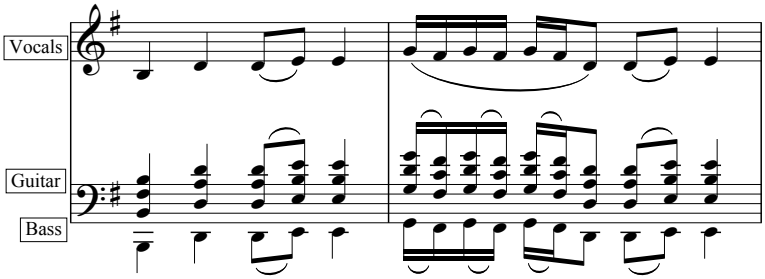
The beginning of the verse of the same song introduces a move from a major versus pentatonic environment to a modal one (example 4.15). The organs chords start, by applying the volume pedal, in such a low volume that they slide to a new mode of C-Aeolian (i.e. parallel minor) so easily that the listener hardly notices the change of mode. Now, the chords and the melody are coherently in the same mode, and support each other. Furthermore, the organ uses voice leading familiar to the practises of *basso continuo* – for instance, there are no parallel fifths in the organ part.




Example 4.15. The beginning of the verse of "July Morning."

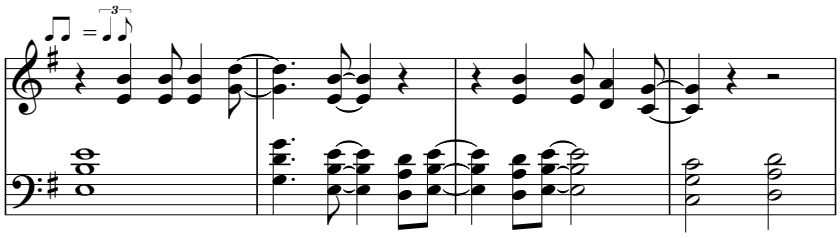
These kinds of fusions between different traditions are quite frequent in heavy metal. The mixtures include not only scales and modes, but voice leading, too. This is dealt with in the following.

Like the modal or scalar practises, the voice leading practises of heavy metal derive from various sources. One quite typical feature of heavy metal, as with most rock music, is parallel voice leading. This means that all the constituents of a harmonic unit (or, chord) are lead in the same direction – either chromatically or diatonically (see, e.g. Persichetti 1961: 198). Especially important in heavy metal are the parallel fifths – the same interval that is banned in all text books on traditional harmony and counterpoint. An extreme example on this is presented by Black Sabbaths "Iron Man" from *Paranoid* (1970) (example 4.16).



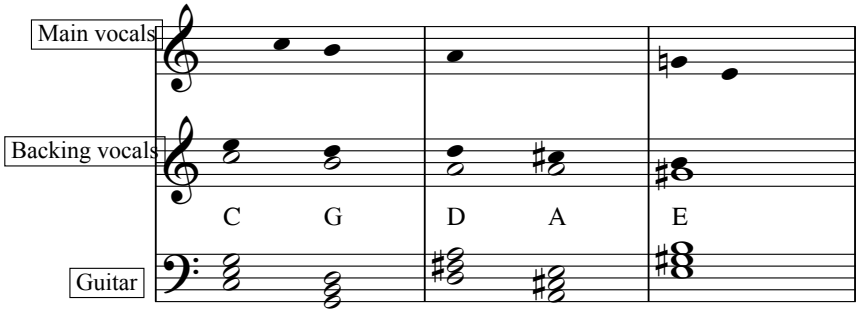

Example 4.16. "Iron Man," the first two bars of the verse.

These kinds of parallel fifths and fourths are also common to the medieval *organum* (e.g. Shepherd 1991: 99; Riemann 1961: 76-77). Still, probably the most important reason for the parallel voice leading is that it is easy to produce all the most common dyads, triads and tetrads with the guitar's *barré* grip, and to move them across the fret board without changing the grip. However, the same type of voice leading can be detected in other instrumental contexts, too (also, see Moore 1995: 191). For instance, also the vocal harmonies to the chorus of Iron Maiden's "Running Free" are executed in parallel motions (example 4.17).



Example 4.17. "Running Free." The vocal harmonies are on the upper, and guitars on the lower stave.

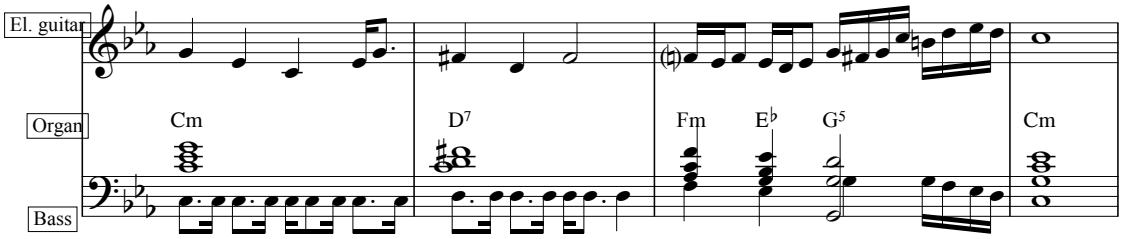
In addition to the parallel voice leading there are more classical continuo-derived practises, too, and quite classical counterpoint between different instrument parts – of course, with the more freely used parallel fifths, unlike in the traditional text book counterpoint (see, e.g. Mann 1971: 82, 86). These features – counterpoint, traditional and parallel voice leading – can occur separately or all together in the same song. For example, in Jimi Hendrix's "Hey Joe" (1983 [1967]) there are all three kinds of motions (example 4.18). The backing vocals form a contrapuntal *obligato* line C-B-A-A-G♯ for the guitar's parallel major triads. After some recapitulations this line is harmonized in such a way that abides by classical voice leading rules. The harmony line starts from the third of the original melody following its descending motion and always hitting the nearest consonance of the next chord. It seems clear that the backing vocal choir is derived from a different tradition than the guitar chords.



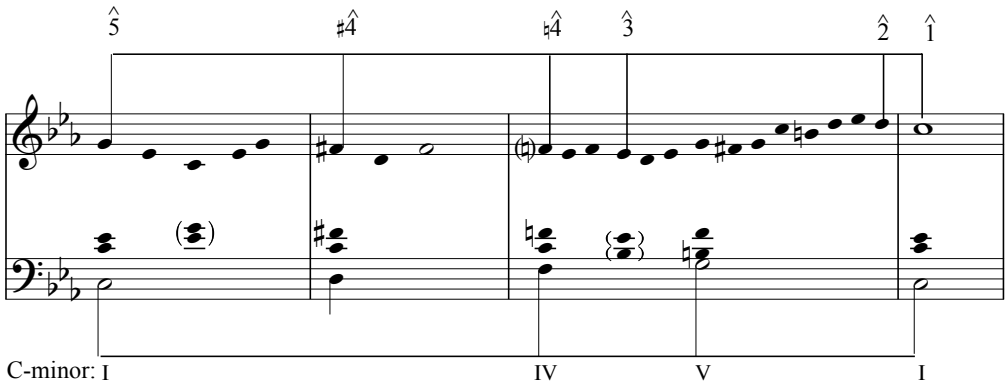

Example 4.18. "Hey Joe." Vocal harmonies on the middle, guitar chords on the bottom stave.

Furthermore, the main vocal line, which is almost the same with the first choir *obligato*, applies a minor third to the last bar giving the song a bluesy pentatonic flavour. Clearly, the song is not pentatonic – the melodies, for instance, are more of the modal type (Aeolian/Dorian/Mixolydian). However, as has been shown, there is interplay of at least three different sources of musical traditions in this song.

In this sense, the instrumental section of "July Morning" is quite different. In fact, it can be analyzed with very traditional tools – even enough so to make a Schenkerian analyst happy (examples 4.19 and 4.20).




Example 4.19. An excerpt of an instrumental section from "July Morning."

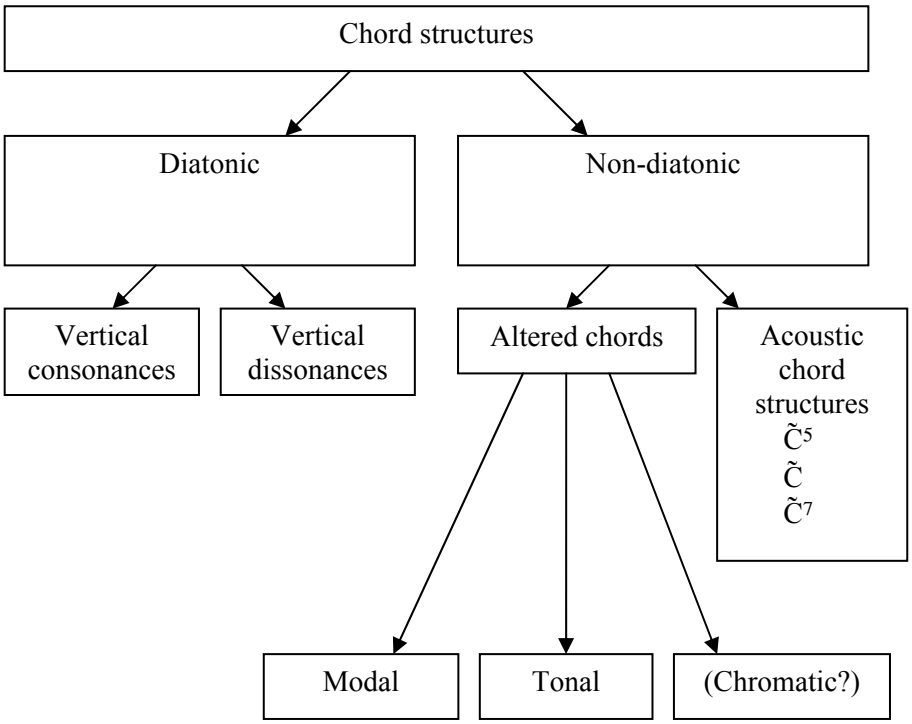



Example 4.20. A Schenkerian type of analysis graph of the previous example.

As shown in example 3.12, there is a fifth-progression (with a chromatic passing not #4) in the melodic line. Moreover, the harmonies can be easily put to satisfy the perfect cadence formula I-IV-V-I. Then the $D^7$ chord and the $E^{\flat}$-major triad have a status of embellishing passing notes. However, there is at least one difficulty in this view. To my mind (or better, to my ears) far too much emphasis is placed on the $\hat{2}$ in the mm. 3, if it is compared to the G in the melody on the same bar. Of course, the D can be heard as filling in the descending line, but as a voice leading act it seems quite artificial. However, for the inner parts this linear approach seems to be more adequate. The organ part actually follows somewhat conventional voice leading. Quite opposite to this, Moore (1995: 190) has stated that "inner parts rarely have linear role, [they are] merely existing to fill out the chord." While this may often be the case for guitar chords such as those heard in "Hey Joe," it seems a somewhat over-generalized view in the light of this example (also, see Wagner 2003: 356n6).

# 5. Chord Structures and Their Relation to Modal/Tonal Context

This chapter deals with typical heavy metal chord structures with the emphasis on their relation to modal and tonal context. Still, other remarks are made when necessary. I have divided heavy metal chords into diatonic and non-diatonic chords, according to their relation to the overall modal scale (example 5.1). Diatonic chords are understood here as chords exclusively employing scale degrees of one single mode, whereas the non-diatonic have at least one scale degree outside of that mode. Non-diatonic chords are subdivided to common altered chords (modal, tonal and chromatic), and chords addressed here as "acoustic."




Example 5.1. The division of chord structures. "C" represents chord a root on any scale degree. The class of chromatic chords is put in parenthesis and labelled with "?" because their relevance will be questioned below.

As Vincent Persichetti (1961: 238) states, "the altered formation [of the chord] must be foreign to the scale presently in effect." Here, altered chords are put into three subcategories according to the type of their alteration. The outline for this subdivision is based on Erkki Salmenhaara's theoretical and analytical writings.[15] The division made by Salmenhaara (1968: 87, 1980: 201-225, 317) is more systematic than, to my knowledge, those made by any other music theorist (e.g. Piston [1962: 276-277], Forte [1962: 96, 355-365]. Persichetti [1961: 237-240] or Schönberg [1966: 211-212, 267-271, 421-431]). Salmenhaara's (1980: 201) *modal altered* chords are those "borrowed" to major or minor environments from the parallel mode (e.g. F-minor chord in C-major is "borrowed" from C-major's parallel mode C-minor). *Tonal altered* chords effect a temporary shift of tonal centre (e.g. secondary dominants, ibid: 212). The rest are called *chromatic altered* chords, and they result from a type of chromatic altering of diatonic chords that cannot be explained by the previous two categories (ibid: 317). Here, while the basic concept of categorizing remains the same, some additional and slightly differing criteria are introduced. For instance, the first two categories are extended in such ways that only a few chords fall into the category of chromatic altered chords. Below, I present my chord categorization and make the necessary remarks on features that distinguish Salmenhaara's division from mine.

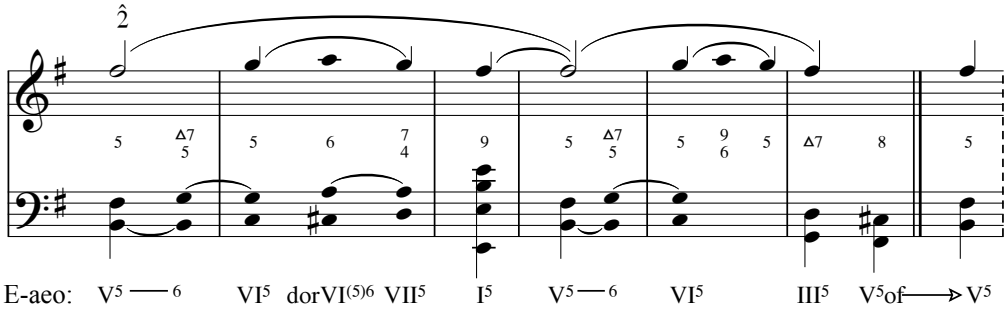
## 5.1. Modal Altered Chords

The first category to fall under the label "altered chords" is the class of modal altered chords. These chords are "borrowed" from some other mode than the present one. In traditional harmony analysis, only possible modes within one tonality are its major and minor forms (e.g. C-major and C-minor). However, by adopting Moore's modal system the boundaries of this category may be expanded. Hence, in the following analyses the concept of modal altering is also extended to the other modes. In analytic labelling, I have used markings derived from Moore's system (see e.g. Moore 2001: 54, 1992: 75-76). In this labelling, the abbreviations of mode names are attached to Roman numerals

[15] Professor Erkki Salmenhaara (1941-2002), a composer and musicologist, was one of Finland's most influential and respected music theorists (see e.g. his obituary in *The Guardian Unlimited* by Guy Rickards, 2002). His treatises *Sointuanalyysi* (Engl. *Harmony Analysis*, 1968) and *Soinnutus* (*Harmonization*, 1980 [1969]) arose mainly from Walter Piston's *Harmony* (1962) whose theories he furthermore developed. *Sointuanalyysi* is still in use as a text book on harmony in most Finnish conservatoires and universities' music departments.

where necessary (e.g., dorIV would denote a triad on the fourth degree of Dorian mode). Markings quite similar to this have been used before (see, e.g. Schenker 1954: 111) – yet, not to the same extent.

In the material studied here, the Aeolian appears to be the most common mode. In most cases, modal altering is really a departure from Aeolian. Quite often an Aeolian chord progression is extended with a chord from Dorian mode. This is the case for instance in Judas Priest's "When the Night Comes Down,"[16] in which the open sixth chord on VI is interpreted as a Dorian loan (example 5.2). That is, an E-Dorian chord in an otherwise E-Aeolian context. The vocal melody, fundamentally kept in $\hat{2}$, locally forms sharp dissonances with chord tones (in the analysis, the major sevenths are especially indicated by $^{\Delta}7$; octave unisons between chord tones and melody are marked if they have any special significance – in the sixth bar, the resolution of the major seventh to octave can be easily shown this way – otherwise they are left out).



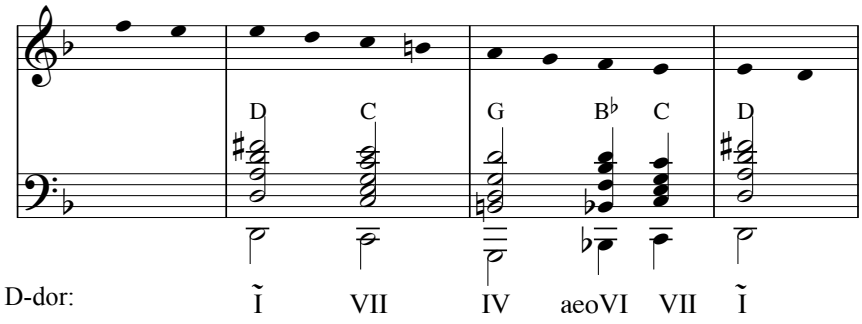

Example 5.2. "When the Night Comes Down," a reduction of the transition-like section just before the guitar solo.

In example 5.2 the chords that form passages from $V^5$ to $I^5$ (mm. 1-3) and from $V^5$ back to $V^5$ (mm. 4-7) are mainly connected in such a way that only one of the two chord tones is moved at a time. The progression from Aeolian $VI^5$ to Dorian $VI^{(5)6}$, and the cadence-like $III^5$-$V^5$ofV are only exceptions to this rule. Dissonances are arranged to occur mainly in off-beats as passing (mm. 1-2 and 4) or neighbour notes (mm. 5). There are only two down-beat dissonances – the major ninth in $I^5$ in mm. 3 that gets its resolution to a chord tone only after the second repetition of the passage (not shown

[16] From the album *Defenders of the Faith*, CBS 1984.

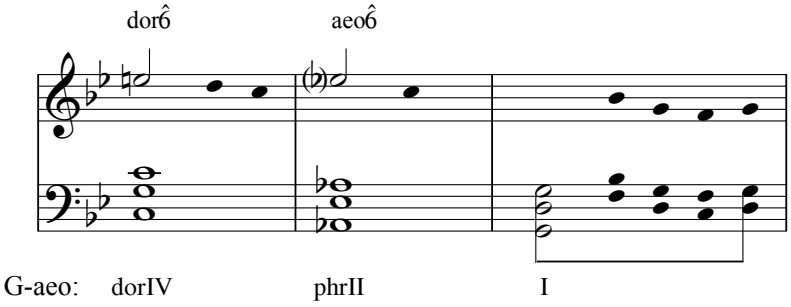
here), and the major seventh in mm. 6, which is resolved by the progression III-VofV that consequently changes it to a consonant chord tone.

The interchange between Dorian and Aeolian can also be detected in Cream's "White Room" (example 5.3). In this case, however, the melodic line strongly underlines D-Dorian. Hence, it is the Aeolian VI that is interpreted as a modal altered chord, or, a modal loan.




Example 5.3. "White Room," verse.

A Dorian $\hat{6}$ occurs in "Smoke on the Water," too (example 5.4). Here the underlining melodic line in the main vocal part – the descending chromatic line from Dorian $\hat{6}$ ($\text{dor}\hat{6}$) to Aeolian $\hat{6}$ ($\text{aeo}\hat{6}$) – is accompanied with Dorian IV (dorIV) and Phrygian II (phrII). The guitar applies only three-note power chords but still the overall harmony is perceived as major triads due to the vocal part.

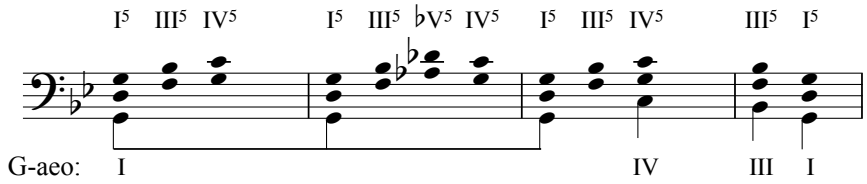



Example 5.4. The chords and the melodic line of the chorus section from "Smoke on the Water."

In terms of traditional harmony analysis, these harmonies are not particularly exceptional – the IV degree from melodic minor and the Neapolitan chord (i.e. major

triad on the flattened second degree). For instance, Salmenhaara (1968: 112) and Piston (1962: 288-289) count the Neapolitan chord as a *chromatic* altered chord, for the Phrygian does not qualify as a major/minor mode according to the music they are dealing with. In these cases, only the major and minor variations of $\hat{3}$ and $\hat{6}$ (and sometimes $\hat{7}$) can pass as modal degrees (see Salmenhaara 1980: 197, 207; 1968: 88-89; Piston 1962: 32; also, see Harrison 1994: 17, and Rameau 1971: 157). However, if we analyse this music from the point of view of modes – as we do here – the phrII is a clear modal altered chord. In fact, this has been done by more traditionally oriented music theorists, too: according to Sadai (1980: 142, 413) and Schoenberg (1967: 33), if the bass descends a half step to a more stable chord, it is addressed as a Phrygian cadence. Furthermore, in his earlier writings, Schenker (1954: 109-110) decides not to use the term "Neapolitan chord," and chooses the "Phrygian II," instead.

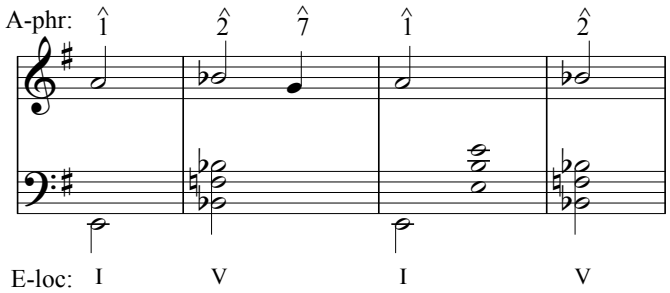
In the case of the guitar riff in "Smoke on the Water," (example 5.5) the ♭V chord can also be interpreted as a modal altered chord (Locrian V).



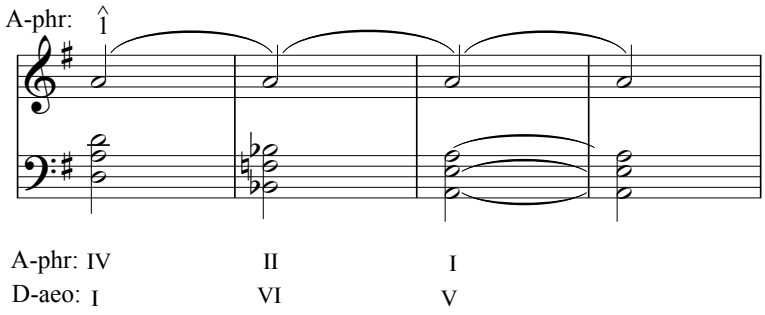

Example 5.5. The riff to "Smoke on the Water."

However, this particular chord calls for an alternative kind of interpretation that maybe should come first. As has been previously stated, the riff may be considered as a single melodic line intensified by parallel fourths below. This melody derives clearly from the so-called blues scale, which is common in e.g. blues derived guitar solos (in this case, the applied scale steps are 1-♭3-4-♭5-4-♭3-1). In addition, the chord progression ♭V-IV is very common in many rock and blues songs based on the twelve bar blues' idiom – in bars 9-10 the ♭V can act as a passing chord between V and IV. Thus, it is of no consequence whether the melodic or harmonic features of the progression are emphasized – the blues-based reading seems to be more accurate.

However, the Locrian interpretation is quite valid for e.g. Black Sabbath's "Symptom of the Universe" (1975) (examples 5.6 and 5.7) and for a great deal of more contemporary heavy metal influenced by Black Sabbath. For example, listen to any song from Metallica's *And Justice for all…* (1988) or Mercyful Fate's *Dead Again* (1998) for these bands the use of the Locrian is quite normative.




Example 5.6. Verse to Black Sabbath: "Symptom of the Universe." Vocal melody is on the upper stave, chords of the guitar riff on the lower. This section is repeated four times.



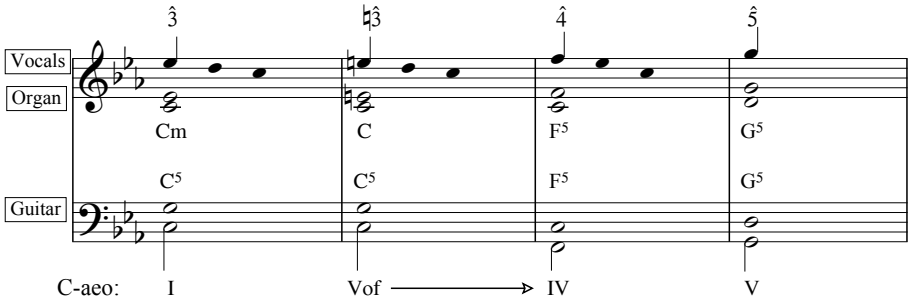

Example 5.7. The chords following the previous example; repeated twice, then back to the previous example 5.6.

"Symptom of the Universe" offers quite an interesting example – the harmonic background established by the guitar riff starting some bars earlier is clearly in E-Locrian, but the vocal melody seems to be in A-Phrygian. The next section (example 5.7) seems to decide in favour of A-Phrygian with a cadence similar to that used in "Smoke on the Water" (example 5.4 above). Some may even hear this in D-Aeolian due to the semitone relationship heard so often between the chords VI and V in minor modes. Still, the melody seems to be persistently in the mode of A-Phrygian – at least I hear it that way. In my opinion, "Symptom" creates a sort of a polymodal effect. For

some this might be proof that this type of analytical method cannot be relevant to this kind of music. However, I suggest that this type of analysis can offer some invaluable insights into the organising of musical structures that may, at face value, seem very simple but are often far more complex.

## 5.2. Tonal Altered Chords

The effect of the *tonal altered* chords is directed towards the tonal centre. In these cases the tonal function of a chord is altered. Usually in the process, some non-tonic chord degree is given the status of a temporary tonal centre (see, e.g. Salmenhaara 1968: 95). This is also near to Schenker's (1954: 256) *tonicalization*. From the point of view of traditional harmony analysis, the most typical tonal altered chords are the secondary dominants – like those in Uriah Heep's "July Morning" (1971) (example 5.8). When the C-minor triad is changed to a C-major its tonal function is changed, too – from the tonic of C-Aeolian to the dominant of F.




Example. 5.8. A secondary dominant in "July Morning."

As often happens in Uriah Heep's music, the Hammond organ fills in the guitar's power chords into triads. Again it is noteworthy that the organist applies quite classical voice leading whereas the guitar is played in parallel motions. Also the melodic line of the vocals is treated in a manner that is common to classical harmony with secondary dominants. This is again an example of the interplay between two traditions – guitar-derived modality and classical contrapuntal voice leading.

However, in heavy metal the secondary subdominants seem to be more common. For instance, the chords of "Hey Joe" follow the cycle of fifths, but in a reversed order, and form the chain of secondary subdominants: IV of IV of IV of IV of I. Similar chains can be found, for example, in Deep Purple's "Hush" (1988 [1968]), "Fireball" (1971), and "Burn" (1974), Rainbow's "Kill the King" (1978), and "Long Live Rock 'n' Roll" (1978), Judas Priest's "Devil's Child" (1982), and Black Sabbath's "A National Acrobat" (1973). Furthermore, all of these are in Aeolian mode.

## 5.3. Acoustic Chords and Chromatic Altered Chords

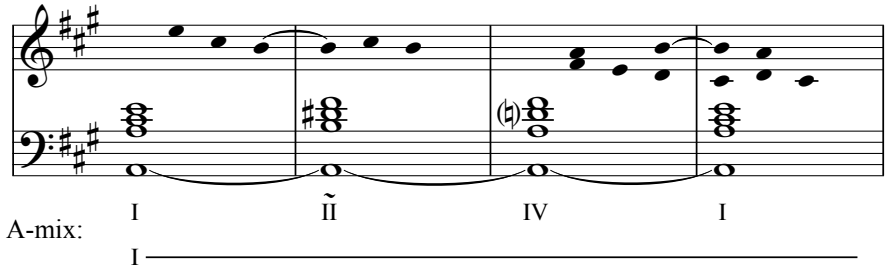
A special interest of this study is in the non-diatonic chords that do not change the mode or a tonal centre, but sort of fuse into a single tone to support the chord's fundamental tone. To the best of my knowledge, these chords have not been subjected to any analytic study. Although Moore (1992: 81) briefly touches on the subject: "rock/pop/soul tends to conceive its harmonies as indivisible units rarely subject to voice leading principles." Yet, no further explanation of these "indivisible units" is given. As far as I know, this is the first effort to systematically take this matter under discussion (for comparison, see Lilja 2001). The structures that I call *Acoustic chords* and the chromatic altered chords are put under the same title here because, as will be shown, these categories are at least overlapping, and sometimes overriding each other.

As discussed in Chapter 2, this category is made of chords that follow the harmonic series. Due to the distortion and high volumes produced by electric guitar the chords, which abide by the lowest partials of the harmonic series, produce much a clearer sound than other chord structures. Depending on the level of distortion, these structures are the power chord, the major triad, and the dominant seventh structures. Actually, these Acoustic chords are not really altered in the sense discussed above, although in their tonal or modal surroundings they may appear to be strange deviations.

For example, the major triad on the first degree in "White Room" (example 5.3 above) is not a secondary dominant – it does not need to resolve, e.g. to the IV degree – but is unambiguously tonic-functioned. Nor it is a modal altered chord – this kind of interpretation would presume that the modal environment changes from fundamentally

minor (Dorian) to major (Mixolydian). The major third of the chord is not perceived as modally significant, but as a vertically important part of the chord's sound. For instance, one can discover the significance of this by playing a D-minor triad instead of this D-major (as sometimes is done in error).

"The Mule" (1971) of Deep Purple has quite similar effect (example 5.9). The excerpt is analyzed in A-Mixolydian because the instrumental section before the sung verse quite clearly establishes the mode with the chords I-VII-I and the melodic tones A-B-C♯-D-E-F♯-G. Analyzing the II degree chord as a Lydian modal altered chord in this context does not quite capture the nature of the chord since the chord's major third is "melted" into the chord structure without establishing a new scale. It would be even harder to interpret it as a secondary dominant, since there is no chord of V. Then the II is more tied to the next chord of IV. Actually, a similar chord progression can be found in "July Morning" (with V, though), but even more so in The Beatles' "Sgt. Pepper's Lonely Hearts Club Band" (1967).

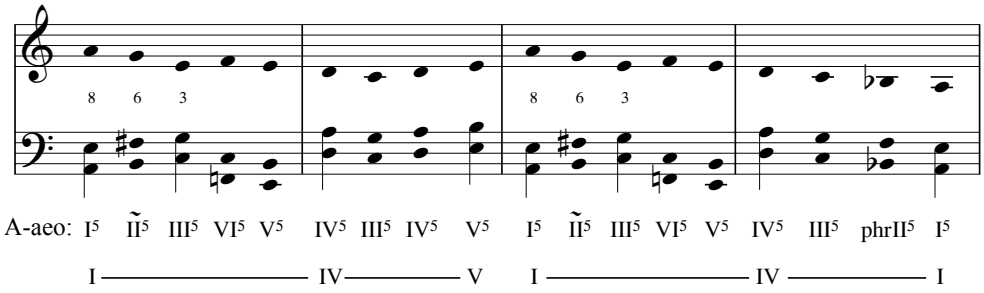



Example 5.9. The verse of "The Mule."

As in "July Morning" these chords in "The Mule" are played on the Hammond organ. However, the acoustic features mentioned in regard to the electric guitar are quite applicable to the Hammond organ, too. Furthermore, this can be taken as an example of how the guitar-derived solutions can be switched to another instrumental context.

The next example will further illuminate on the difference between a modal altered and an Acoustic chord. This excerpt is from the "outro" riff to Dio's "Shame on the Night" (example 5.10). Apparently, the passing chord $II^5$ between the I and the III degree includes an A-Dorian altered tone (F♯). However, the F♯, while forming a perfect fifth

with a chord root B, is so well fused into the fundamental that it has completely lost its independence. In fact, the G of the vocal choir should form a harsh dissonance with F♯, but this is not aurally present at all – actually, the listener notices only the minor sixth between the B and the G.



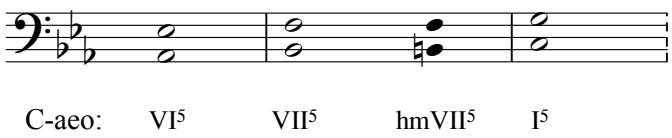

Example 5.10. The ending guitar riff to "Shame on the Night." The guitar part on the lower stave and the vocal choir on the upper.

The only actually altered chord here is the Phrygian II degree, where the modal altering is directed to the chord root – hence, it is the only one of the chords where a new scalar content is explicitly present.

As stated before, these Acoustic chords may seem to be strange in their contexts, but often are explainable by their vertical appearance. When considering heavy metal chords, it seems that the vertical direction is often more important than the horizontal one. In other words, the clear sounding of a chord is more valuable than following the prevailing mode. However, these chords are usually connected to the modal and tonal context via the root. Clearly, their construction is not diatonic, yet they act alike, and are used as such.

*Chromatic Altered Chords?*

Next I will present the issues concerning the supposed class of chromatic altered chords. Tonal and modal altered chords are moved from another tonality or mode to some other environment. Chromatic altered chords are made by altering the chords of a key (Salmenhaara 1980: 317). If compared to those of Salmenhaara, the previous two categories are extended in such a way that it diminishes the number of chromatic altered chords. For instance, the category of modal altered chords is much wider than Salmenhaara's (1968: 88-89) due to the modal system. However, depending on the point of view some chords could be placed to this category. For instance, a diminished chord that sometimes occurs in Aeolian context could be analyzed as a chromatically altered formation. This is done, for example, in Dio's "Holy Diver" (1983) (example 5.11).

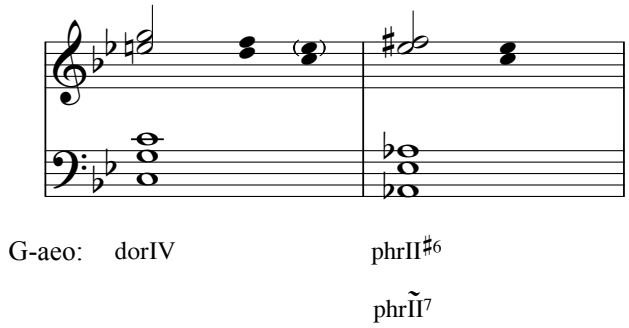



Example 5.11. The dyad of the diminished fifth in "Holy Diver."

Although the diminished fifth is here presented as an alien formation to Aeolian mode, it can be interpreted as a modal altered chord. Of course, this requires that the Harmonic minor (abbr. hm) is accepted as a modal scale. If we recognise the influence of classical music in heavy metal, this seems reasonable enough. Furthermore, in Western music as a whole the harmonic minor is one of the most common scales. However, in the larger scale analysis these kinds of intervallic structures are usually interpreted as chromatic passing chords. Still, as a single unit it is not a chromatic but a modal chord, because it can be found in several modes. Even though, now the relation between the Aeolian mode and the Harmonic minor mode is much more accurate – since the Aeolian is much more common in heavy metal, the harmonies abiding by Harmonic minor should be (in most cases) interpreted as altered formations.

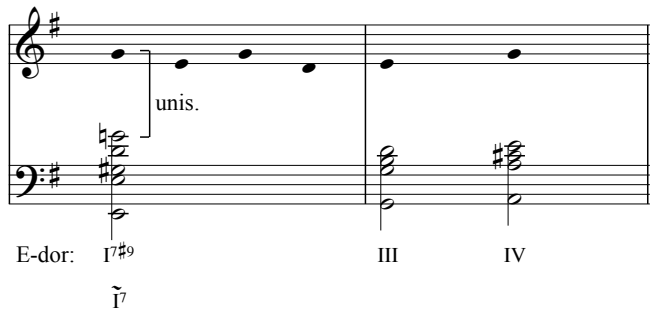
In the following, there are further examples of seemingly chromatic chords with their likely interpretations. One of these is the harmonic formation between the chord and

the vocal harmonies in the chorus of "Smoke on the Water" (example 5.12). Apparently, the harmonies form a chord of an augmented sixth that is often interpreted as a chromatic altered chord (by, e.g. Forte 1962: 356-361; Salmenhaara 1980: 317). The chord of the augmented sixth does not occur in any of the modes, so by no means, can this chord on a Phrygian II degree be interpreted as a chromatic altered chord phrII$^{\sharp 6}$. On the other hand – when recalling the points made in Chapter 2 – it is possible to interpret the chord enharmonically as a Phrygian II degree that is harmonized with an Acoustic dominant seventh structure, i.e. phr$\tilde{\text{II}}^{7}$.




Example 5.12. The chorus of "Smoke on the Water" with vocal harmonies.

In the next example, there is a chord that cannot be found in any of the previously mentioned diatonic seven-note modes. This is the 7♯9 chord, especially favoured by Jimi Hendrix, the chord that includes the major and the minor thirds (example 5.13). In spite of the apparent dominant structure, the E$^{7\sharp 9}$ in "Purple Haze" is unambiguously a tonic chord – it is hard to imagine anyone to expect it to resolve a fourth above, or to interpret it in a dominant function.

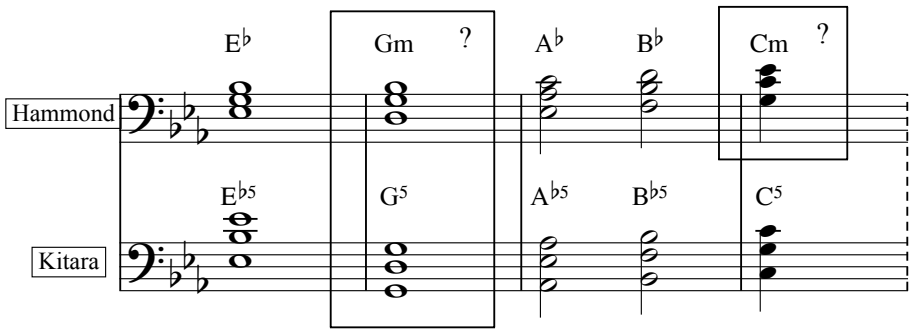



Example 5.13. The beginning of the verse of "Purple Haze."

The $E^{7\sharp 9}$ can be interpreted as a chromatic altered chord $I^{7\sharp 9}$, but still, there is another perspective to this. The four lowest tones of the chord form an acoustic dominant seventh structure $\tilde{I}^{7}$ that supports the fundamental E. In terms of harmony analysis this means that, in a sense, there is only a single tone (E) that is supported by the structure abiding by its harmonic series. Furthermore, according to this interpretation the chord's minor third G is a part of the minor pentatonic scale of the melody – the guitarist supports the melody with his instrument.

## 5.4. Diatonic Vertical Dissonances

Since the diatonic vertical consonances are self-explanatory (i.e. they follow both the mode and the acoustic structure), this chapter will deal with diatonic *vertical dissonances*. These chords follow the prevailing mode, but are acoustically difficult in a way that was discussed in Chapter 2 – with the distortion and high volumes their construction cause dissonant effects. For instance, all the chords of the chorus of "July Morning" are diatonic (example 5.14).



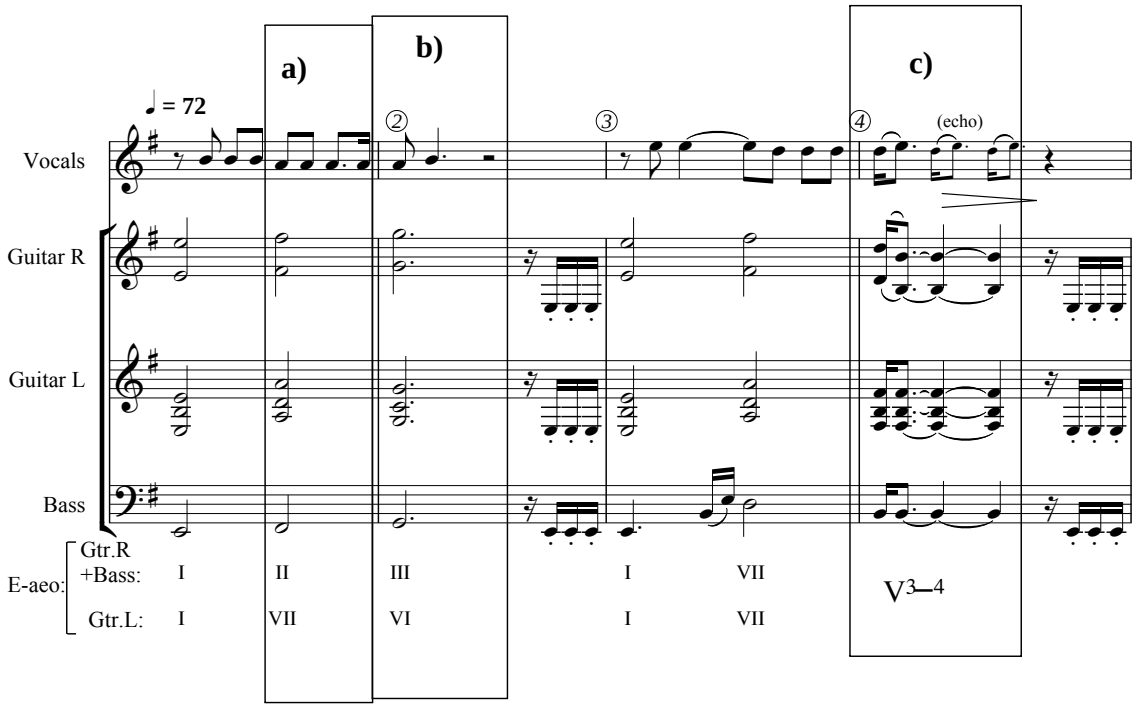

Example 5.14. The chorus section of "July Morning."

The organist fills in the guitar's power chords into diatonic triads. However, if the chords are explored aurally, it can be noticed that the Gm triad is considerably more unstable than the other chords. I suggest that the imbalance of this chord is due to its minor formation. The minor third does not support the harmonics of G, but causes different harmonic series instead, and thus considerable beating, too. This is further

emphasized due to the guitar's power chords that strongly support the harmonics of G. The reason why the Cm triad on the last bar does not cause the same kind of effect is probably that the chord is rhythmically interrupted before the rise of a different harmonic series. This excerpt can be conveniently listened to by using the panorama potentiometer to listen to one channel at a time – first the right channel with Hammond only, then both channels to get the guitar chords, too. As mentioned above, it seems that the similar kinds of effects as those of the guitar are raised by the Hammond organ. Of course, when listening to these phenomena one has to keep in mind that only a high enough volume will do justice to the harmonics.

According to the chord categorisation presented here, all loud and distorted harmonic structures that do not abide by the harmonic series of the root tone are vertical dissonances. Thus, the open-minor-sixth dyad belongs to this category, too. By listening to the aforementioned examples (e.g. "The Sentinel") it is possible to detect the dissonance of the sixth when compared to e.g. the power chord of the fifth. More examples on the open-minor-sixth can be heard in e.g. the following songs: AC/DC: "Highway to Hell" (1979); Dio: "Holy Diver" (1983), "Gypsy" (1983), and "One Night in the City" (1984); Iron Maiden: "The Number of the Beast" (1982).

The phenomenon of vertical dissonance is very clear in a contemporary example – Mercyful Fate's "Banshee" (1998) (example 5.15).




Example 5.15. An excerpt of the chorus section of "Banshee."

When listening to this example, there appear to be some strange dissonances in the places marked with a) and b). However, when notated, they appear to be quite consonant in a traditional sense. If the left and right channels are listened to separately this controversy could be explained as follows: The bass and the vocals are sounding in both channels. In the right channel, there is only one guitar sounding (marked as "Guitar R" in the example) that is playing in unison with the bass. The right-channel guitar and the bass with vocals refer to E-Aeolian chord degrees I, II, and III. On the other hand, the guitar on the left channel (Guitar L) plays power chords on the I, VII, and VI degree. In a) this results a II degree chord in its first inversion, i.e. the intervals of a minor third and a minor sixth. According to the acoustic features of loud and distorted chords discussed in Chapter 2, both of these are categorized as vertical dissonances by definition due to their intervallic structure. This can be easily compared to a similar chord in mm. 2, where all the instruments support the VII (D). It is significant that the bass supports the harmonics of D – due to this, the D-chord of mm. 2 does not beat in the way that the "D-chord" does in mm. 1.

The situation in b) is slightly different. Here all except the left-channel guitar form a III degree chord. The left-channel guitar's VI creates two dissonances – not only a major

seventh with the vocals but also a low fourth with the bass. This fourth (G-C) creates a harmonic series of C, but it is suppressed by the series of G that is emphasized by all the other instruments, and thus, left with little weight – still, enough to create a fuzzy sound. "Banshee" includes yet another interesting phenomenon. In c) the vocal line clearly reaches the tonic E, yet the guitars and the bass are in the V degree. This creates a sort of reversed suspended-fourth effect ($V^{3\text{-}4}$). Actually, in this case we can hear two separate tonal functions (the tonic and the dominant) without them sounding at all controversial or dissonant.

The perspectives presented here give some explanation of the frequent avoidance of vertical dissonances in heavy metal music. There are several ways of doing this – one being the use of Acoustic chords in spite of tonal or modal context. However, if the dissonant structures are used, they can be e.g. arpeggiated to reduce the vertical beating – then the beating frequencies are not played together. This is done in, for example Judas Priest's "When the Night Comes Down" where a minor triad is played with arpeggios, although with heavy distortion. Furthermore, it seems likely that the increasing amount of distortion has resulted in a dropping of the thirds altogether – when the distortion grows, the beating resulting from the harmonics of the equal-tempered major third will rise accordingly. The tendency to use only or mainly dyads has been clear in the run up to the 1980's and up to the present day. Of course, often these dissonances are applied for musical purposes – the alternation between consonance and dissonance is really a basic element of any melodic or harmonic construction. Nevertheless, there is often a decision to be made (consciously or unconsciously) whether to use a vertical or a modal/tonal dissonance.

# 6. Conclusions

What has been done here is a musicological survey of heavy metal chord structures from various points of view. The study has covered their acoustic constructions, their relation to traditional music analysis, and their most influential musical sources in history. Furthermore, the chord categorizing principles were used as an instrument of music theory to discuss the chords' relations to their environments. In the light of this study, heavy metal music has proved to be far more complicated than has been previously suggested (cf. Everett 2000: 330-335).

The acoustic characteristics of heavy metal chords have not been studied before to this extent – for instance, Walser (1993: 43) devoted only a single paragraph to the acoustics. Yet, these acoustic features bring a distinctive sound to the music, and have a great deal of influence on the perception of apparently (when notated) simple formations. Furthermore, it seems that the perspectives presented here can explain the frequent avoidance of the vertically dissonant structures in heavily distorted metal music. Neither can we ignore the fact that heavy metal music derives from various musical traditions. The classical major/minor tonal influences are often paralleled with pentatonic blues-derived contents, as indeed they are with the modal scales. Thus, the music in question is an explicitly multi-traditional one. It is quite interesting (and perhaps surprising) that heavy metal harmony often tends to use solutions that were established in the modal music of the Renaissance and furthermore fixed during the Baroque era. In heavy metal, this kind of chord connecting is coupled with pentatonic and blues-derived features (although, mostly within the equal-tempered system).

However, this study leaves many questions unanswered. For instance, the acoustics of heavy metal chords has to be subjected to further, empirical studies. In fact, the acoustic measuring of the chords will be one of my next objects of study, in addition to the way in which musicians and the listeners perceive the chord roots and fundamental tones. These studies should involve at least spectral analysis and the methods applied for the studies of musical cognition. Also, the functional tonal hierarchies of heavy metal chord progressions have to be further explored (although this has been done extensively in Lilja 2002).

Furthermore, the analyses made here have emphasized the vertical aspects at the expense of the horizontal, although some features were explored with regard to voice leading (e.g. voice leading analysis of "July Morning"). The horizontal aspects will be covered more thoroughly in the next phase of my study on heavy metal harmony. In fact, the analyses made here are the first step to a multi-level analytic model that should be capable of dealing with the music in several hierarchical levels – from the details to the overall "fundamental" structures. To accomplish this I will take two major theories as starting points – the Schenkerian and Riemannian – and explore their possibilities while using heavy metal as an analytic corpus. I strongly believe in the need for a detailed analysis of heavy metal music. Armed with a deeper understanding of the nuances of the music itself, it will be easier to embark on larger scale analyses of all aspects of heavy metal music; from the musical to the social and cultural perspectives and onwards.

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**Appendix I**

Some combination tones of the A-major triad with justly tempered intervals.
While some differences come out negative it is not relevant to this study – as in Arthur Benade's study (1976: 257) "it is the *magnitude* of the frequency difference that matters to us here."

| $f_2$-$f_1$ | | $f_1$ | $2\,f_1$ | $3\,f_1$ | $4\,f_1$ | $5\,f_1$ | $6\,f_1$ | $7\,f_1$ | $8\,f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_2$ | 165 | 55 | -55 | -165 | -275 | -385 | -495 | -605 | -715 |
| $2\,f_2$ | 330 | 220 | 110 | 0 | -110 | -220 | -330 | -440 | -550 |
| $3\,f_2$ | 495 | 385 | 275 | 165 | 55 | -55 | -165 | -275 | -385 |
| $4\,f_2$ | 660 | 550 | 440 | 330 | 220 | 110 | 0 | -110 | -220 |
| $5\,f_2$ | 825 | 715 | 605 | 495 | 385 | 275 | 165 | 55 | -55 |
| $6\,f_2$ | 990 | 880 | 770 | 660 | 550 | 440 | 330 | 220 | 110 |
| $7\,f_2$ | 1155 | 1045 | 935 | 825 | 715 | 605 | 495 | 385 | 275 |
| $8\,f_2$ | 1320 | 1210 | 1100 | 990 | 880 | 770 | 660 | 550 | 440 |

| $f_3$-$f_1$ | | $f_1$ | $2\,f_1$ | $3\,f_1$ | $4\,f_1$ | $5\,f_1$ | $6\,f_1$ | $7\,f_1$ | $8\,f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_3$ | 220 | 110 | 0 | -110 | -220 | -330 | -440 | -550 | -660 |
| $2\,f_3$ | 440 | 330 | 220 | 110 | 0 | -110 | -220 | -330 | -440 |
| $3\,f_3$ | 660 | 550 | 440 | 330 | 220 | 110 | 0 | -110 | -220 |
| $4\,f_3$ | 880 | 770 | 660 | 550 | 440 | 330 | 220 | 110 | 0 |
| $5\,f_3$ | 1100 | 990 | 880 | 770 | 660 | 550 | 440 | 330 | 220 |
| $6\,f_3$ | 1320 | 1210 | 1100 | 990 | 880 | 770 | 660 | 550 | 440 |
| $7\,f_3$ | 1540 | 1430 | 1320 | 1210 | 1100 | 990 | 880 | 770 | 660 |
| $8\,f_3$ | 1760 | 1650 | 1540 | 1430 | 1320 | 1210 | 1100 | 990 | 880 |

| $f_4$- $f_1$ | | $f_1$ | $2\,f_1$ | $3\,f_1$ | $4\,f_1$ | $5\,f_1$ | $6\,f_1$ | $7\,f_1$ | $8\,f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_4$ | 275 | 165 | 55 | -55 | -165 | -275 | -385 | -495 | -605 |
| $2\,f_4$ | 550 | 440 | 330 | 220 | 110 | 0 | -110 | -220 | -330 |
| $3\,f_4$ | 825 | 715 | 605 | 495 | 385 | 275 | 165 | 55 | -55 |
| $4\,f_4$ | 1100 | 990 | 880 | 770 | 660 | 550 | 440 | 330 | 220 |
| $5\,f_4$ | 1375 | 1265 | 1155 | 1045 | 935 | 825 | 715 | 605 | 495 |
| $6\,f_4$ | 1650 | 1540 | 1430 | 1320 | 1210 | 1100 | 990 | 880 | 770 |
| $7\,f_4$ | 1925 | 1815 | 1705 | 1595 | 1485 | 1375 | 1265 | 1155 | 1045 |
| $8\,f_4$ | 2200 | 2090 | 1980 | 1870 | 1760 | 1650 | 1540 | 1430 | 1320 |

| $f_3$- $f_2$ | | $f_2$ | $2\,f_2$ | $3\,f_2$ | $4\,f_2$ | $5\,f_2$ | $6\,f_2$ | $7\,f_2$ | $8\,f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 165 | 330 | 495 | 660 | 825 | 990 | 1155 | 1320 |
| $f_3$ | 220 | 55 | -110 | -275 | -440 | -605 | -770 | -935 | -1100 |
| $2f_3$ | 440 | 275 | 110 | -55 | -220 | -385 | -550 | -715 | -880 |
| $3\,f_3$ | 660 | 495 | 330 | 165 | 0 | -165 | -330 | -495 | -660 |
| $4\,f_3$ | 880 | 715 | 550 | 385 | 220 | 55 | -110 | -275 | -440 |
| $5\,f_3$ | 1100 | 935 | 770 | 605 | 440 | 275 | 110 | -55 | -220 |
| $6\,f_3$ | 1320 | 1155 | 990 | 825 | 660 | 495 | 330 | 165 | 0 |
| $7\,f_3$ | 1540 | 1375 | 1210 | 1045 | 880 | 715 | 550 | 385 | 220 |
| $8\,f_3$ | 1760 | 1595 | 1430 | 1265 | 1100 | 935 | 770 | 605 | 440 |

| $f_4$- $f_2$ | | $f_2$ | 2 $f_2$ | 3 $f_2$ | 4 $f_2$ | 5 $f_2$ | 6 $f_2$ | 7 $f_2$ | 8 $f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 165 | 330 | 495 | 660 | 825 | 990 | 1155 | 1320 |
| $f_4$ | 275 | 110 | -55 | -220 | -385 | -550 | -715 | -880 | -1045 |
| 2 $f_4$ | 550 | 385 | 220 | 55 | -110 | -275 | -440 | -605 | -770 |
| 3 $f_4$ | 825 | 660 | 495 | 330 | 165 | 0 | -165 | -330 | -495 |
| 4 $f_4$ | 1100 | 935 | 770 | 605 | 440 | 275 | 110 | -55 | -220 |
| 5 $f_4$ | 1375 | 1210 | 1045 | 880 | 715 | 550 | 385 | 220 | 55 |
| 6 $f_4$ | 1650 | 1485 | 1320 | 1155 | 990 | 825 | 660 | 495 | 330 |
| 7 $f_4$ | 1925 | 1760 | 1595 | 1430 | 1265 | 1100 | 935 | 770 | 605 |
| 8 $f_4$ | 2200 | 2035 | 1870 | 1705 | 1540 | 1375 | 1210 | 1045 | 880 |

| $f_4$- $f_3$ | | $f_3$ | 2 $f_3$ | 3 $f_3$ | 4 $f_3$ | 5 $f_3$ | 6 $f_3$ | 7 $f_3$ | 8 $f_3$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 220 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 |
| $f_4$ | 275 | 55 | -165 | -385 | -605 | -825 | -1045 | -1265 | -1485 |
| 2 $f_4$ | 550 | 330 | 110 | -110 | -330 | -550 | -770 | -990 | -1210 |
| 3 $f_4$ | 825 | 605 | 385 | 165 | -55 | -275 | -495 | -715 | -935 |
| 4 $f_4$ | 1100 | 880 | 660 | 440 | 220 | 0 | -220 | -440 | -660 |
| 5 $f_4$ | 1375 | 1155 | 935 | 715 | 495 | 275 | 55 | -165 | -385 |
| 6 $f_4$ | 1650 | 1430 | 1210 | 990 | 770 | 550 | 330 | 110 | -110 |
| 7 $f_4$ | 1925 | 1705 | 1485 | 1265 | 1045 | 825 | 605 | 385 | 165 |
| 8 $f_4$ | 2200 | 1980 | 1760 | 1540 | 1320 | 1100 | 880 | 660 | 440 |

| $f_2$+$f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_2$ | 165 | 275 | 385 | 495 | 605 | 715 | 825 | 935 | 1045 |
| 2 $f_2$ | 330 | 440 | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 |
| 3 $f_2$ | 495 | 605 | 715 | 825 | 935 | 1045 | 1155 | 1265 | 1375 |
| 4 $f_2$ | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 | 1430 | 1540 |
| 5 $f_2$ | 825 | 935 | 1045 | 1155 | 1265 | 1375 | 1485 | 1595 | 1705 |
| 6 $f_2$ | 990 | 1100 | 1210 | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 |
| 7 $f_2$ | 1155 | 1265 | 1375 | 1485 | 1595 | 1705 | 1815 | 1925 | 2035 |
| 8 $f_2$ | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 | 1980 | 2090 | 2200 |

| $f_3$+$f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_3$ | 220 | 330 | 440 | 550 | 660 | 770 | 880 | 990 | 1100 |
| 2 $f_3$ | 440 | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 |
| 3 $f_3$ | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 | 1430 | 1540 |
| 4 $f_3$ | 880 | 990 | 1100 | 1210 | 1320 | 1430 | 1540 | 1650 | 1760 |
| 5 $f_3$ | 1100 | 1210 | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 | 1980 |
| 6 $f_3$ | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 | 1980 | 2090 | 2200 |
| 7 $f_3$ | 1540 | 1650 | 1760 | 1870 | 1980 | 2090 | 2200 | 2310 | 2420 |
| 8 $f_3$ | 1760 | 1870 | 1980 | 2090 | 2200 | 2310 | 2420 | 2530 | 2640 |

| $f_4 + f_1$ | | $f_1$ | $2\,f_1$ | $3\,f_1$ | $4\,f_1$ | $5\,f_1$ | $6\,f_1$ | $7\,f_1$ | $8\,f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_4$ | 275 | 385 | 495 | 605 | 715 | 825 | 935 | 1045 | 1155 |
| $2\,f_4$ | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 | 1430 |
| $3\,f_4$ | 825 | 935 | 1045 | 1155 | 1265 | 1375 | 1485 | 1595 | 1705 |
| $4\,f_4$ | 1100 | 1210 | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 | 1980 |
| $5\,f_4$ | 1375 | 1485 | 1595 | 1705 | 1815 | 1925 | 2035 | 2145 | 2255 |
| $6\,f_4$ | 1650 | 1760 | 1870 | 1980 | 2090 | 2200 | 2310 | 2420 | 2530 |
| $7\,f_4$ | 1925 | 2035 | 2145 | 2255 | 2365 | 2475 | 2585 | 2695 | 2805 |
| $8\,f_4$ | 2200 | 2310 | 2420 | 2530 | 2640 | 2750 | 2860 | 2970 | 3080 |

| $f_3 + f_2$ | | $f_2$ | $2\,f_2$ | $3\,f_2$ | $4\,f_2$ | $5\,f_2$ | $6\,f_2$ | $7\,f_2$ | $8\,f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 165 | 330 | 495 | 660 | 825 | 990 | 1155 | 1320 |
| $f_3$ | 220 | 385 | 550 | 715 | 880 | 1045 | 1210 | 1375 | 1540 |
| $2f_3$ | 440 | 605 | 770 | 935 | 1100 | 1265 | 1430 | 1595 | 1760 |
| $3\,f_3$ | 660 | 825 | 990 | 1155 | 1320 | 1485 | 1650 | 1815 | 1980 |
| $4\,f_3$ | 880 | 1045 | 1210 | 1375 | 1540 | 1705 | 1870 | 2035 | 2200 |
| $5\,f_3$ | 1100 | 1265 | 1430 | 1595 | 1760 | 1925 | 2090 | 2255 | 2420 |
| $6\,f_3$ | 1320 | 1485 | 1650 | 1815 | 1980 | 2145 | 2310 | 2475 | 2640 |
| $7\,f_3$ | 1540 | 1705 | 1870 | 2035 | 2200 | 2365 | 2530 | 2695 | 2860 |
| $8\,f_3$ | 1760 | 1925 | 2090 | 2255 | 2420 | 2585 | 2750 | 2915 | 3080 |

| $f_4 + f_2$ | | $f_2$ | $2\,f_2$ | $3\,f_2$ | $4\,f_2$ | $5\,f_2$ | $6\,f_2$ | $7\,f_2$ | $8\,f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 165 | 330 | 495 | 660 | 825 | 990 | 1155 | 1320 |
| $f_4$ | 275 | 440 | 605 | 770 | 935 | 1100 | 1265 | 1430 | 1595 |
| $2\,f_4$ | 550 | 715 | 880 | 1045 | 1210 | 1375 | 1540 | 1705 | 1870 |
| $3\,f_4$ | 825 | 990 | 1155 | 1320 | 1485 | 1650 | 1815 | 1980 | 2145 |
| $4\,f_4$ | 1100 | 1265 | 1430 | 1595 | 1760 | 1925 | 2090 | 2255 | 2420 |
| $5\,f_4$ | 1375 | 1540 | 1705 | 1870 | 2035 | 2200 | 2365 | 2530 | 2695 |
| $6\,f_4$ | 1650 | 1815 | 1980 | 2145 | 2310 | 2475 | 2640 | 2805 | 2970 |
| $7\,f_4$ | 1925 | 2090 | 2255 | 2420 | 2585 | 2750 | 2915 | 3080 | 3245 |
| $8\,f_4$ | 2200 | 2365 | 2530 | 2695 | 2860 | 3025 | 3190 | 3355 | 3520 |

| $f_4 + f_3$ | | $f_3$ | $2\,f_3$ | $3\,f_3$ | $4\,f_3$ | $5\,f_3$ | $6\,f_3$ | $7\,f_3$ | $8\,f_3$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 220 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 |
| $f_4$ | 275 | 495 | 715 | 935 | 1155 | 1375 | 1595 | 1815 | 2035 |
| $2\,f_4$ | 550 | 770 | 990 | 1210 | 1430 | 1650 | 1870 | 2090 | 2310 |
| $3\,f_4$ | 825 | 1045 | 1265 | 1485 | 1705 | 1925 | 2145 | 2365 | 2585 |
| $4\,f_4$ | 1100 | 1320 | 1540 | 1760 | 1980 | 2200 | 2420 | 2640 | 2860 |
| $5\,f_4$ | 1375 | 1595 | 1815 | 2035 | 2255 | 2475 | 2695 | 2915 | 3135 |
| $6\,f_4$ | 1650 | 1870 | 2090 | 2310 | 2530 | 2750 | 2970 | 3190 | 3410 |
| $7\,f_4$ | 1925 | 2145 | 2365 | 2585 | 2805 | 3025 | 3245 | 3465 | 3685 |
| $8\,f_4$ | 2200 | 2420 | 2640 | 2860 | 3080 | 3300 | 3520 | 3740 | 3960 |

**Appendix II**

Some combination tones of the A-major triad with equal-tempered intervals

| $f_2$-$f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_2$ | 165,82 | 55,82 | -54,18 | -164,18 | -274,18 | -384,18 | -494,18 | -604,18 | -714,18 |
| 2 $f_2$ | 331,64 | 221,64 | 111,64 | 1,64 | -108,36 | -218,36 | -328,36 | -438,36 | -548,36 |
| 3 $f_2$ | 497,46 | 387,46 | 277,46 | 167,46 | 57,46 | -52,54 | -162,54 | -272,54 | -382,54 |
| 4 $f_2$ | 663,28 | 553,28 | 443,28 | 333,28 | 223,28 | 113,28 | 3,28 | -106,72 | -216,72 |
| 5 $f_2$ | 829,1 | 719,1 | 609,1 | 499,1 | 389,1 | 279,1 | 169,1 | 59,1 | -50,9 |
| 6 $f_2$ | 994,92 | 884,92 | 774,92 | 664,92 | 554,92 | 444,92 | 334,92 | 224,92 | 114,92 |
| 7 $f_2$ | 1160,74 | 1050,74 | 940,74 | 830,74 | 720,74 | 610,74 | 500,74 | 390,74 | 280,74 |
| 8 $f_2$ | 1326,56 | 1216,56 | 1106,56 | 996,56 | 886,56 | 776,56 | 666,56 | 556,56 | 446,56 |

| $f_3$-$f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_3$ | 220 | 110 | 0 | -110 | -220 | -330 | -440 | -550 | -660 |
| 2 $f_3$ | 440 | 330 | 220 | 110 | 0 | -110 | -220 | -330 | -440 |
| 3 $f_3$ | 660 | 550 | 440 | 330 | 220 | 110 | 0 | -110 | -220 |
| 4 $f_3$ | 880 | 770 | 660 | 550 | 440 | 330 | 220 | 110 | 0 |
| 5 $f_3$ | 1100 | 990 | 880 | 770 | 660 | 550 | 440 | 330 | 220 |
| 6 $f_3$ | 1320 | 1210 | 1100 | 990 | 880 | 770 | 660 | 550 | 440 |
| 7 $f_3$ | 1540 | 1430 | 1320 | 1210 | 1100 | 990 | 880 | 770 | 660 |
| 8 $f_3$ | 1760 | 1650 | 1540 | 1430 | 1320 | 1210 | 1100 | 990 | 880 |

| $f_4$- $f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_4$ | 277 | 167 | 57 | -53 | -163 | -273 | -383 | -493 | -603 |
| 2 $f_4$ | 554 | 444 | 334 | 224 | 114 | 4 | -106 | -216 | -326 |
| 3 $f_4$ | 831 | 721 | 611 | 501 | 391 | 281 | 171 | 61 | -49 |
| 4 $f_4$ | 1108 | 998 | 888 | 778 | 668 | 558 | 448 | 338 | 228 |
| 5 $f_4$ | 1385 | 1275 | 1165 | 1055 | 945 | 835 | 725 | 615 | 505 |
| 6 $f_4$ | 1662 | 1552 | 1442 | 1332 | 1222 | 1112 | 1002 | 892 | 782 |
| 7 $f_4$ | 1939 | 1829 | 1719 | 1609 | 1499 | 1389 | 1279 | 1169 | 1059 |
| 8 $f_4$ | 2216 | 2106 | 1996 | 1886 | 1776 | 1666 | 1556 | 1446 | 1336 |

| $f_3$- $f_2$ | | $f_2$ | 2 $f_2$ | 3 $f_2$ | 4 $f_2$ | 5 $f_2$ | 6 $f_2$ | 7 $f_2$ | 8 $f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 164,82 | 329,64 | 494,46 | 659,28 | 824,1 | 988,92 | 1153,74 | 1318,56 |
| $f_3$ | 220 | 55,18 | -109,64 | -274,46 | -439,28 | -604,1 | -768,92 | -933,74 | -1098,56 |
| 2$f_3$ | 440 | 275,18 | 110,36 | -54,46 | -219,28 | -384,1 | -548,92 | -713,74 | -878,56 |
| 3 $f_3$ | 660 | 495,18 | 330,36 | 165,54 | 0,72 | -164,1 | -328,92 | -493,74 | -658,56 |
| 4 $f_3$ | 880 | 715,18 | 550,36 | 385,54 | 220,72 | 55,9 | -108,92 | -273,74 | -438,56 |
| 5 $f_3$ | 1100 | 935,18 | 770,36 | 605,54 | 440,72 | 275,9 | 111,08 | -53,74 | -218,56 |
| 6 $f_3$ | 1320 | 1155,18 | 990,36 | 825,54 | 660,72 | 495,9 | 331,08 | 166,26 | 1,44 |
| 7 $f_3$ | 1540 | 1375,18 | 1210,36 | 1045,54 | 880,72 | 715,9 | 551,08 | 386,26 | 221,44 |
| 8 $f_3$ | 1760 | 1595,18 | 1430,36 | 1265,54 | 1100,72 | 935,9 | 771,08 | 606,26 | 441,44 |

| $f_4$- $f_2$ | | $f_2$ | 2 $f_2$ | 3 $f_2$ | 4 $f_2$ | 5 $f_2$ | 6 $f_2$ | 7 $f_2$ | 8 $f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 164,82 | 329,64 | 494,46 | 659,28 | 824,1 | 988,92 | 1153,74 | 1318,56 |
| $f_4$ | 277 | 112,18 | -52,64 | -217,46 | -382,28 | -547,1 | -711,92 | -876,74 | -1041,56 |
| 2 $f_4$ | 554 | 389,18 | 224,36 | 59,54 | -105,28 | -270,1 | -434,92 | -599,74 | -764,56 |
| 3 $f_4$ | 831 | 666,18 | 501,36 | 336,54 | 171,72 | 6,9 | -157,92 | -322,74 | -487,56 |
| 4 $f_4$ | 1108 | 943,18 | 778,36 | 613,54 | 448,72 | 283,9 | 119,08 | -45,74 | -210,56 |
| 5 $f_4$ | 1385 | 1220,18 | 1055,36 | 890,54 | 725,72 | 560,9 | 396,08 | 231,26 | 66,44 |
| 6 $f_4$ | 1662 | 1497,18 | 1332,36 | 1167,54 | 1002,72 | 837,9 | 673,08 | 508,26 | 343,44 |
| 7 $f_4$ | 1939 | 1774,18 | 1609,36 | 1444,54 | 1279,72 | 1114,9 | 950,08 | 785,26 | 620,44 |
| 8 $f_4$ | 2216 | 2051,18 | 1886,36 | 1721,54 | 1556,72 | 1391,9 | 1227,08 | 1062,26 | 897,44 |

| $f_4$- $f_3$ | | $f_3$ | 2 $f_3$ | 3 $f_3$ | 4 $f_3$ | 5 $f_3$ | 6 $f_3$ | 7 $f_3$ | 8 $f_3$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 220 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 |
| $f_4$ | 277 | 57 | -163 | -383 | -603 | -823 | -1043 | -1263 | -1483 |
| 2 $f_4$ | 554 | 334 | 114 | -106 | -326 | -546 | -766 | -986 | -1206 |
| 3 $f_4$ | 831 | 611 | 391 | 171 | -49 | -269 | -489 | -709 | -929 |
| 4 $f_4$ | 1108 | 888 | 668 | 448 | 228 | 8 | -212 | -432 | -652 |
| 5 $f_4$ | 1385 | 1165 | 945 | 725 | 505 | 285 | 65 | -155 | -375 |
| 6 $f_4$ | 1662 | 1442 | 1222 | 1002 | 782 | 562 | 342 | 122 | -98 |
| 7 $f_4$ | 1939 | 1719 | 1499 | 1279 | 1059 | 839 | 619 | 399 | 179 |
| 8 $f_4$ | 2216 | 1996 | 1776 | 1556 | 1336 | 1116 | 896 | 676 | 456 |

| $f_2$+$f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_2$ | 165,82 | 275,82 | 385,82 | 495,82 | 605,82 | 715,82 | 825,82 | 935,82 | 1045,82 |
| 2 $f_2$ | 331,64 | 441,64 | 551,64 | 661,64 | 771,64 | 881,64 | 991,64 | 1101,64 | 1211,64 |
| 3 $f_2$ | 497,46 | 607,46 | 717,46 | 827,46 | 937,46 | 1047,46 | 1157,46 | 1267,46 | 1377,46 |
| 4 $f_2$ | 663,28 | 773,28 | 883,28 | 993,28 | 1103,28 | 1213,28 | 1323,28 | 1433,28 | 1543,28 |
| 5 $f_2$ | 829,1 | 939,1 | 1049,1 | 1159,1 | 1269,1 | 1379,1 | 1489,1 | 1599,1 | 1709,1 |
| 6 $f_2$ | 994,92 | 1104,92 | 1214,92 | 1324,92 | 1434,92 | 1544,92 | 1654,92 | 1764,92 | 1874,92 |
| 7 $f_2$ | 1160,74 | 1270,74 | 1380,74 | 1490,74 | 1600,74 | 1710,74 | 1820,74 | 1930,74 | 2040,74 |
| 8 $f_2$ | 1326,56 | 1436,56 | 1546,56 | 1656,56 | 1766,56 | 1876,56 | 1986,56 | 2096,56 | 2206,56 |

| $f_3$+$f_1$ | | $f_1$ | 2 $f_1$ | 3 $f_1$ | 4 $f_1$ | 5 $f_1$ | 6 $f_1$ | 7 $f_1$ | 8 $f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_3$ | 220 | 330 | 440 | 550 | 660 | 770 | 880 | 990 | 1100 |
| 2 $f_3$ | 440 | 550 | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 |
| 3 $f_3$ | 660 | 770 | 880 | 990 | 1100 | 1210 | 1320 | 1430 | 1540 |
| 4 $f_3$ | 880 | 990 | 1100 | 1210 | 1320 | 1430 | 1540 | 1650 | 1760 |
| 5 $f_3$ | 1100 | 1210 | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 | 1980 |
| 6 $f_3$ | 1320 | 1430 | 1540 | 1650 | 1760 | 1870 | 1980 | 2090 | 2200 |
| 7 $f_3$ | 1540 | 1650 | 1760 | 1870 | 1980 | 2090 | 2200 | 2310 | 2420 |
| 8 $f_3$ | 1760 | 1870 | 1980 | 2090 | 2200 | 2310 | 2420 | 2530 | 2640 |

| $f_{4+}f_1$ | | $f_1$ | $2f_1$ | $3f_1$ | $4f_1$ | $5f_1$ | $6f_1$ | $7f_1$ | $8f_1$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 110 | 220 | 330 | 440 | 550 | 660 | 770 | 880 |
| $f_4$ | 277 | 387 | 497 | 607 | 717 | 827 | 937 | 1047 | 1157 |
| $2f_4$ | 554 | 664 | 774 | 884 | 994 | 1104 | 1214 | 1324 | 1434 |
| $3f_4$ | 831 | 941 | 1051 | 1161 | 1271 | 1381 | 1491 | 1601 | 1711 |
| $4f_4$ | 1108 | 1218 | 1328 | 1438 | 1548 | 1658 | 1768 | 1878 | 1988 |
| $5f_4$ | 1385 | 1495 | 1605 | 1715 | 1825 | 1935 | 2045 | 2155 | 2265 |
| $6f_4$ | 1662 | 1772 | 1882 | 1992 | 2102 | 2212 | 2322 | 2432 | 2542 |
| $7f_4$ | 1939 | 2049 | 2159 | 2269 | 2379 | 2489 | 2599 | 2709 | 2819 |
| $8f_4$ | 2216 | 2326 | 2436 | 2546 | 2656 | 2766 | 2876 | 2986 | 3096 |

| $f_{3+}f_2$ | | $f_2$ | $2f_2$ | $3f_2$ | $4f_2$ | $5f_2$ | $6f_2$ | $7f_2$ | $8f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 164,82 | 329,64 | 494,46 | 659,28 | 824,1 | 988,92 | 1153,74 | 1318,56 |
| $f_3$ | 220 | 384,82 | 549,64 | 714,46 | 879,28 | 1044,1 | 1208,92 | 1373,74 | 1538,56 |
| $2f_3$ | 440 | 604,82 | 769,64 | 934,46 | 1099,28 | 1264,1 | 1428,92 | 1593,74 | 1758,56 |
| $3f_3$ | 660 | 824,82 | 989,64 | 1154,46 | 1319,28 | 1484,1 | 1648,92 | 1813,74 | 1978,56 |
| $4f_3$ | 880 | 1044,82 | 1209,64 | 1374,46 | 1539,28 | 1704,1 | 1868,92 | 2033,74 | 2198,56 |
| $5f_3$ | 1100 | 1264,82 | 1429,64 | 1594,46 | 1759,28 | 1924,1 | 2088,92 | 2253,74 | 2418,56 |
| $6f_3$ | 1320 | 1484,82 | 1649,64 | 1814,46 | 1979,28 | 2144,1 | 2308,92 | 2473,74 | 2638,56 |
| $7f_3$ | 1540 | 1704,82 | 1869,64 | 2034,46 | 2199,28 | 2364,1 | 2528,92 | 2693,74 | 2858,56 |
| $8f_3$ | 1760 | 1924,82 | 2089,64 | 2254,46 | 2419,28 | 2584,1 | 2748,92 | 2913,74 | 3078,56 |

| $f_{4+}f_2$ | | $f_2$ | $2f_2$ | $3f_2$ | $4f_2$ | $5f_2$ | $6f_2$ | $7f_2$ | $8f_2$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 164,82 | 329,64 | 494,46 | 659,28 | 824,1 | 988,92 | 1153,74 | 1318,56 |
| $f_4$ | 277 | 441,82 | 606,64 | 771,46 | 936,28 | 1101,1 | 1265,92 | 1430,74 | 1595,56 |
| $2f_4$ | 554 | 718,82 | 883,64 | 1048,46 | 1213,28 | 1378,1 | 1542,92 | 1707,74 | 1872,56 |
| $3f_4$ | 831 | 995,82 | 1160,64 | 1325,46 | 1490,28 | 1655,1 | 1819,92 | 1984,74 | 2149,56 |
| $4f_4$ | 1108 | 1272,82 | 1437,64 | 1602,46 | 1767,28 | 1932,1 | 2096,92 | 2261,74 | 2426,56 |
| $5f_4$ | 1385 | 1549,82 | 1714,64 | 1879,46 | 2044,28 | 2209,1 | 2373,92 | 2538,74 | 2703,56 |
| $6f_4$ | 1662 | 1826,82 | 1991,64 | 2156,46 | 2321,28 | 2486,1 | 2650,92 | 2815,74 | 2980,56 |
| $7f_4$ | 1939 | 2103,82 | 2268,64 | 2433,46 | 2598,28 | 2763,1 | 2927,92 | 3092,74 | 3257,56 |
| $8f_4$ | 2216 | 2380,82 | 2545,64 | 2710,46 | 2875,28 | 3040,1 | 3204,92 | 3369,74 | 3534,56 |

| $f_{4+}f_3$ | | $f_3$ | $2f_3$ | $3f_3$ | $4f_3$ | $5f_3$ | $6f_3$ | $7f_3$ | $8f_3$ |
|---|---|---|---|---|---|---|---|---|---|
| | | 220 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 |
| $f_4$ | 277 | 497 | 717 | 937 | 1157 | 1377 | 1597 | 1817 | 2037 |
| $2f_4$ | 554 | 774 | 994 | 1214 | 1434 | 1654 | 1874 | 2094 | 2314 |
| $3f_4$ | 831 | 1051 | 1271 | 1491 | 1711 | 1931 | 2151 | 2371 | 2591 |
| $4f_4$ | 1108 | 1328 | 1548 | 1768 | 1988 | 2208 | 2428 | 2648 | 2868 |
| $5f_4$ | 1385 | 1605 | 1825 | 2045 | 2265 | 2485 | 2705 | 2925 | 3145 |
| $6f_4$ | 1662 | 1882 | 2102 | 2322 | 2542 | 2762 | 2982 | 3202 | 3422 |
| $7f_4$ | 1939 | 2159 | 2379 | 2599 | 2819 | 3039 | 3259 | 3479 | 3699 |
| $8f_4$ | 2216 | 2436 | 2656 | 2876 | 3096 | 3316 | 3536 | 3756 | 3976 |